# Identification of the PEA Hysteresis Property Using a Minimum Variance Scheme

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***Abstract*—The minimum variance principle is, generally, used for controller's synthesis. In this paper, we propose another use of this principle to identify the hysteresis property of piezoelectric actuators using the Extended Least Squares identification technique adapted for the ARMAX model.**

***IndexTerms*—Piezoelectric actuator, nonlinearity, hysteresis, identification.**

## I. Introduction

The piezoelectric actuators (PEA) based on the inverse piezoelectric effect are used in many fields due to their properties. Indeed, for example they are very used in the ultra-precision applications [1]-[3]. However, the hysteresis property, existing in piezoelectric materials, makes the modeling and the control of PEA difficult. Many nonlinear models was developed in the literature to describe the hysteresis property of piezoelectric actuators such as the Preisach model and its modifications [4]-[8], the Duhem model [9], [10], the Maxwell Resistance Capacitor (MRC) model [11], the Bouc-Wen model [12]-[15], the Prandtl-Ishlinskii model [16]-[20] and the modified Rayleigh model [21]. A survey on these models can be found in [22]. Furthermore, the experimentation showed that the hysteresisnon-linearity in PEA is not symmetric and many models was proposed in [23]-[25] to describe the asymmetric hysteresis existing in PEA. To compensate the hysteresis behavior of PEA, many intelligent techniques was used such as fuzzy logic [26], [27], neural networks [28], [29], adaptive filter [30], [31], hybrid models [32], NARMAX models [33], [34] and iterative learning control [35]. The most previous models are nonlinear and difficult to implement in on-line which makes the controller synthesis and analysis difficult. To deal with this problem, the PEA can be described by linear models using identification algorithms [36], [37]. In this paper, we propose a technique for the description of the hysteresis property. This technique is based on the modification of the minimum variance controller algorithm to be used for identification purpose. This paper is organized as follows: the extended least squares recursive identification method is described in Section II, then, the proposed minimum variance identification scheme is presented in Section III and before concluding, the proposed approach is validated through simulation results.

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## II. The Extended Least Squares (ELS) Identification Methods

The piezoelectric actuator can be described by the ARMAX model of the following expression:

$$A(q^{-1})y(t) = q^{-d}B(q^{-1})u(t) + C(q^{-1})e(t) \tag{1}$$

with

$$A(q^{-1}) = 1 + \sum_{i=1}^{n_A} a_i q^{-i}$$

$$B(q^{-1}) = \sum_{i=1}^{n_B} b_i q^{-i}$$

$$C(q^{-1}) = 1 + \sum_{i=1}^{n_C} c_i q^{-i}$$

And *y(t), u(t)* are respectively the output and the input signals, *e(t)* is a white noise with zero mean value and constant variance and d is pure time delay.

Hence, model (1) can be written as:

$$\begin{aligned} y(t+1) &= -\sum_{i=1}^{n_A} a_i y(t+1-i) + \sum_{i=1}^{n_B} b_i u(t+1-d-i) \\ &+ e(t+1) + \sum_{i=1}^{n_C} c_i e(t+1-i) = \theta^T \varphi(t) + e(t+1) \end{aligned} \tag{2}$$

$$\theta^T = [a_1, \ldots, a_{n_A}, b_1, \ldots, b_{n_B}, c_1, \ldots, c_{n_C}]$$

$$\varphi^T = [-y(t), \ldots, -y(t+1-n_A), u(t-d), \ldots, u(t+1-d-n_B), e(t), \ldots, e(t+1-n_C)]$$

If we assume that the estimate of $\theta$ is $\Theta$, the estimated output $y$ is given by:

$$y(t+1) = \theta^T(t)\varphi(t) \tag{3}$$

where $\theta^T = [a_1, \ldots, a_{n_A}, \hat{b}_1, \ldots, \hat{b}_{n_B}, \hat{c}_1, \ldots, \hat{c}_{n_C}]$

The prediction error between the real and the estimated output $\varepsilon(t)$ is defined by:

$$\varepsilon(t) = y(t+1) - y(t+1) = y(t+1) - \theta^T(t)\varphi(t) \tag{4}$$

We define also the criterion $J(t)$:

$$\min_{\theta(t)} J(t) = \frac{1}{2}\sum_{i=1}^{t}\left[ y(i) - \theta^T(t)\varphi(i-1) \right]^2 \tag{5}$$

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Minimizing this criterion means:

$$\frac{\partial J(t)}{\partial \theta(t)} = \sum_{i=1}^{t}\left(-\varphi(i-1)\left[y(i)-\theta^{T}(t)\varphi(i-1)\right]\right)^{T} = 0 \qquad (6)$$

From (6), the estimated parameters vector $\hat{\theta}$ is obtained as follows:

$$\theta(t) = \left(\sum_{i=1}^{t}\varphi(i-1)\varphi^{T}(i-1)\right)^{-1}\left[\sum_{i=1}^{t}y(i)\varphi(i-1)\right] \qquad (7)$$

We put:

$$\theta(t) = F(t)\sum_{i=1}^{t}y(i)\varphi(i-1) \qquad (8)$$

With

$$F^{-1}(t) = \sum_{i=1}^{t}\varphi(i-1)\varphi^{T}(i-1) \qquad (9)$$

From (9):

$$F^{-1}(t) = \sum_{i=1}^{t+1}\varphi(i-1)\varphi^{T}(i-1) = \sum_{i=1}^{t}\varphi(i-1)\varphi^{T}(i-1)+\varphi(t)\varphi^{T}(t)$$
$$= F^{-1}(t)+\varphi(t)\varphi^{T}(t)$$

Thus, the adaptation gain $F(t+1)$ is given by:

$$F(t+1) = F(t)-\frac{F(t)\varphi(t)\varphi^{T}(t)F(t)}{1+\varphi^{T}(t)F(t)\varphi(t)} \qquad (10)$$

From (8):

$$\sum_{i=1}^{t}y(i)\varphi(i-1) = F^{-1}(t)\theta(t) \qquad (11)$$

For $i = 1,\ldots, t+1$, we have:

$$\sum_{i=1}^{t+1}y(i)\varphi(i-1) = \sum_{i=1}^{t}y(i)\varphi(i-1)+y(t+1)\varphi(t)$$
$$= F^{-1}(t)\theta(t)+y(t+1)\varphi(t) \qquad (12)$$

From (11) and (12) we have:

$$F^{-1}(t+1)\theta(t+1) = F^{-1}(t)\theta(t)+y(t+1)\varphi(t)$$
$$= F^{-1}(t)\theta(t+1)+\varphi(t)\varepsilon(t+1)$$

We obtain:

$$\theta(t+1) = \theta(t)+F(t+1)\varphi(t)\varepsilon(t+1) \qquad (13)$$

where

$$F(t+1) = F(t)-\frac{F(t)\varphi(t)\varphi^{T}(t)F(t)}{1+\varphi^{T}(t)F(t)\varphi(t)} \qquad (14)$$

And

$\theta(t)$ : Estimated parameters vector

$F(t)$ : Adaptation gain

$\varepsilon(t)$ : Prediction error

$\varphi(t)$ : Observations vector

Equations (13) and (14) are called parametric adaptation algorithm (PAA) which is used for all recursive identification techniques. To validate the identified model, the estimations of the normalized autocorrelations $RN(i)$ are calculated as follows:

$$RN(i) = \frac{R(i)}{R(0)}; i = 1,\ldots,\max(n_A, n_B+d) \qquad (15)$$

With

$$R(i) = \frac{1}{N}\sum_{t=1}^{N}\varepsilon(t)\varepsilon(t-i); R(0) = \frac{1}{N}\sum_{t=1}^{N}\varepsilon^{2}(t) \qquad (16)$$

The identified model is valid if $|RN(i)|<0.15$ [38].

## III. Description of the Minimum Variance Identification Scheme

In this paper we modified the minimum variance controller scheme to be used for identification as shown in Fig. 1. Indeed, instead of using as input a reference signal, we use the estimated output. In the diagram of Fig. 1 to be used for identification purpose, $y(t)$ is the system output, $u(t)$ is the control signal and $y(t+1+d)$ is the predicted output. $R$ and $S$ are the regulator polynomials and are given by [39], [40]:

$$R(q^{-1}) = \sum_{i=1}^{n_R}r_i q^{-i}$$

$$S(q^{-1}) = \sum_{i=1}^{n_S}S_i q^{-i}$$

where $n_R = n_B+d+1$

$n_S = n_A-1$

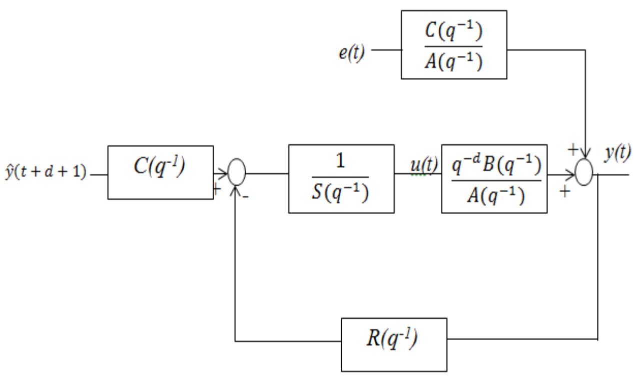



Fig. 1. Proposed minimum variance identification scheme.

From the diagram we have:

$$C(q^{-1})y(t+d+1) = R(q^{-1})y(t)+S(q^{-1})u(t) \qquad (17)$$

For the case $d = 0$, we have

$$y(t+1) = -\sum_{i=1}^{n_C} c_i\, y(t+1-i) + R(q^{-1})y(t) + S(q^{-1})u(t)$$
$$= \theta^T \varphi(t) \qquad (18)$$

Then, the PAA of the equations (13) and (14) can be applied with

$$\theta^T = [\hat{r}_0, \ldots, \hat{r}_{n_R}, \hat{s}_0, \ldots, \hat{s}_{n_S}, \hat{c}_1, \ldots, \hat{c}_{n_C}]$$

$$\varphi^T = [y(t), \ldots, y(t-n_R), u(t-d), \ldots, u(t-n_S), y(t), \ldots, y(t-n_C)]$$

## IV. Simulation Results

In this section, we use the algorithms of equations (13) and (14) with the parameters and observations vectors of the proposed approach to identify the hysteresis property of the piezoelectric actuator.

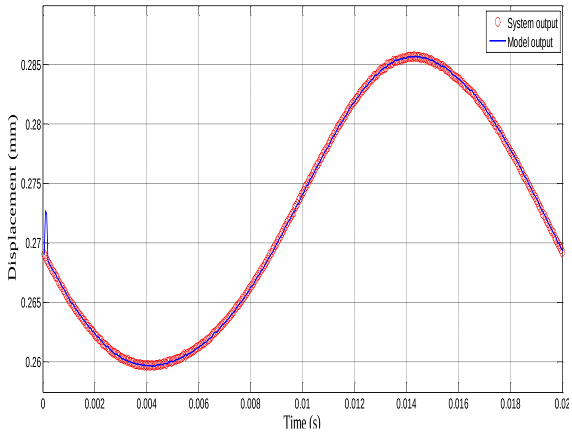
In order to validate the proposed approach, we use the Matlab environment to implement the different algorithms. The data file (I/O file) used for the identification corresponds to the piezoelectric actuator APA-120ML excited by a sinusoidal signal of frequency 50Hz:

$$u(t) = 68.5\sin(2\_\times 50t+0.44) + 61.5(v).$$

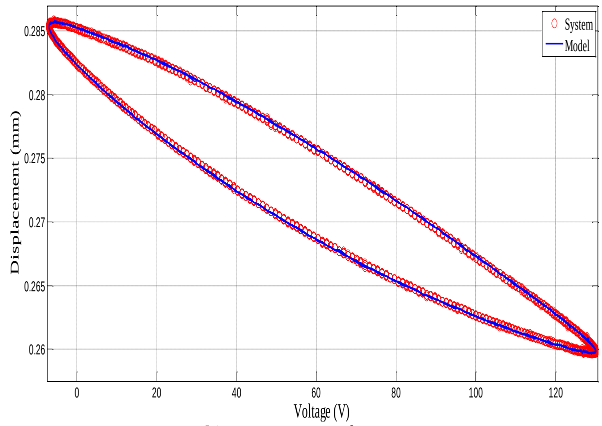
The piezoelectric actuators are generally modeled by a second order system, therefore the polynomials orders of the model are selected as follows: $n_A = 2$, $n_B = 2$, $n_C = 1$ and $d = 0$. which corresponds to $n_R = 1$, $n_S = 1$ and $n_C = 1$.

The identification results and the convergence of the regulator parameters are shown in Fig. 2a, Fig. 2b, and Fig. 2c. According to Fig. 2a, we can remark that the proposed approach can ensure satisfactory results with an identified behavior of PEA close to the original one.

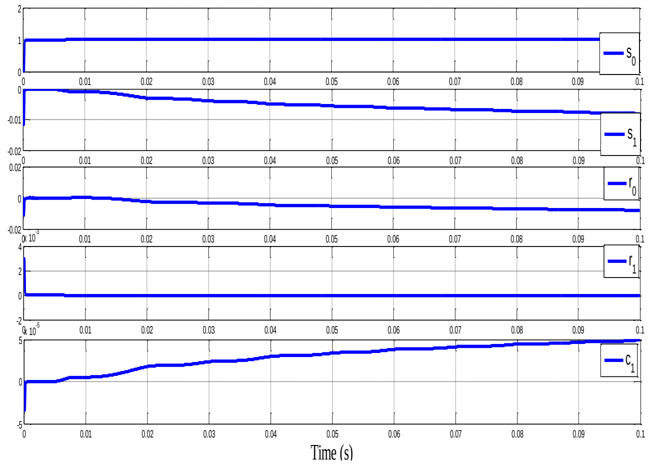
To evaluate proposed approach performances, the identification relative error and the values of its autocorrelation function are given in Fig. 3a and Fig. 3b, respectively. These figures shows that the maximum relative error of identification is 5%, also $|RN(i)|$ are all small than 0.15. Therefore, the prediction error tends towards a white noise which validates and draws clearly the performance of the proposed approach.



(a) Outputs comparison.

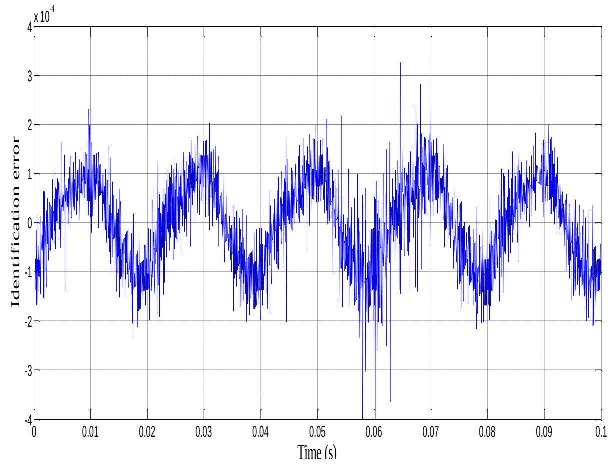


(b) Input-Output feature.

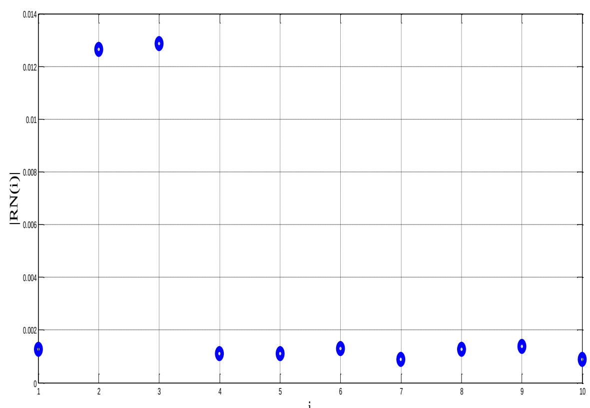


(c) Variation of the regulator parameters.

Fig. 2. Identification results.



(a) Identification error.



(b) Error autocorrelation.

Fig. 3. Validation of the technique.

## V. Conclusion

In this paper, an identification scheme was proposed based on the minimum variance principle to identify the hysteresis property existing in PEA. An on-line model is constructed based on the ARMAX model and the extended least squares algorithm is used to represent this phenomenon.

Simulation results validated and showed the performance of the proposed approach. Furthermore, they showed also that the ARMAX model can characterize well the hysteresis non linearity in piezoelectric actuators which facilitates the analysis and the control of these devices.

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