International Journal of Computer and Electrical Engineering, Vol. 1, No. 5 December, 2009
1793-8163

Fuzzy Complex System of Linear Equations Applied to Circuit Analysis

Taher Rahgooy, Hadi Sadoghi Yazdi, Reza Monsefi

Abstract— **In this paper, application of fuzzy system of linear equations (FSLE) is investigated in the circuit analysis (CA). In the field of CA, each circuit includes linear resistance, inductance and capacitance are modeled to complex linear equation. A fuzzy current or voltage in the circuit is more intellectual relative to crisp value because of some reasons as environmental conditions, tolerance in the elements, and noise (leakage of power harmonics,). This scheme forces for presentation of fuzzy complex system of linear equations (FCSLE). Derivation is presented for solving FCSLE and obtained results show ability of this method in CA.**

Index Terms— **Fuzzy complex system of linear equations, Fuzzy system of linear equations, Circuit analysis, Fuzzy current and voltage, Noise, Tolerance.**

I. Introduction

Wide range of real world applications in many areas including economics, engineering and physics, are using linear system of equations for modeling and solving their respective problems. In many situations the estimation of the system parameters is imprecise and only some vague knowledge about the actual value of parameters is available, therefore to overcome this problem the vagueness of parameters is represented by fuzzy numbers. In many applications some of the parameters are usually represented by complex numbers which have fuzzy nature, thus it is important to develop mathematical models that would appropriately treat fuzzy complex linear systems. Circuits can be modeled in the form of fuzzy complex system of linear equations (FCSLE).A rapid growth of interest in circuit theory and simulation is seen in recent decade. A circuit simulation program (e.g., SPICES [1] and QPMDFSD [2, 3, and 4]) provides very good details in simulation of circuits. These types of software provide various facilities for obtaining best answer which is close to real form. Some contributions have been made in stability theory using functional analysis in the study of nonlinear systems, and computer-aided nonlinear network analysis [5, 6, 7, and 8]. But uncertainty in circuit parameters and environmental conditions lead to develop a new method which takes in to account uncertainty in circuit analysis. In the previous work [9], we considered circuit equation as a fuzzy differential equation (FDE) with fuzzy variables. We applied two conventional types of fuzzy derivative in circuit analysis. In [9] simple circuits were investigated and transient response were elicited. In present study steady state analysis of complex circuit is investigated from FSLE perspective. This scheme forces for presentation of fuzzy complex system of linear equations (FCSLE). Therefore in the following, related works on FSLE field is presented.

Hadi Sadoghi Yazdi is with the Computer Department, Ferdowsi University of Mashhad, Mashhad, Iran, (sadoghi@sttu.ac.ir) (Corresponding Author)

Reza Monsefi is with the Computer Department, Ferdowsi University of Mashhad, Mashhad, Iran, (rmonsefi@um.ac.ir)

II. Related works

A particular type of fuzzy systems, $Ax = b$, where the coefficient matrix A is crisp, while b is a fuzzy number vector, is investigated by Friedman et al. [10], Ma et al. [15] and Allahviranloo[11–13], by using interval analysis techniques. Friedman et al. [10] has used the embedding method given in [19], they replaced the original fuzzy linear system with a crisp linear system having a nonnegative coefficient matrix S, which may be singular even if A is nonsingular. Allahviranloo [11–13] uses the iterative Jacobi and Gauss Siedel method, the Adomian method and the Successive over-relaxation method, respectively. Dehghan and Hashemi [14, 20] extended several well-known numerical algorithms such as Richardson, Extrapolated Richardson, Jacobi, JOR, Gauss–Seidel, EGS, SOR, AOR, ESOR, SSOR, USSOR, EMA and MSOR to solve system of linear equations. Ma et al. [15], starting from the work by Friedman et al. [10], analyzed the solution of fuzzy systems of the form $A_1x = A_2x + b$. They remarked that the system $A_1x = A_2x + b$ is not equivalent to the system $(A_1 - A_2) x = b$, since for an arbitrary fuzzy number u there exists no element v such that $u + v = 0$. They introduced the conditions under which the new system had a solution. Muzziolia et al [16] discussed fuzzy linear systems in the form of $A_1x + b_1 = A_2x + b_2$ with A_1, A_2 square matrices of fuzzy coefficients and b_1, b_2 fuzzy number vectors. They clarified the link between interval linear systems and fuzzy linear systems also a generalization of the vector solution of Buckley and Qu [17]. Abbasbandy and A. Jafarian 2006 proposed the steepest descent method, for solving FSLE [18]. In the aforementioned works, real coefficients are discussed but in many applications as circuit analysis, while SLE includes complex coefficients and complex variables. Also we encountered situations in circuit solving that needs solving a FSLE model which has been solved by Friedman et al. [10]. Therefore we present a Fuzzy Complex SLE, namely, FCSLE in this paper. This model works well in circuit solving. Major notes in this paper are presentation of FCSLE, and application of FSLE and FCSLE in circuit solving. Organization of this paper is as follows:

FSLE is explained in section III. Section IV appropriate to the proposed FCSLE and experimental results over circuit analysis are discussed in the section V. Final section includes the conclusion.

III. Fuzzy System of Linear Equations

A. Preliminaries

We represented an arbitrary fuzzy number by an ordered pair of functions $(\underline{u}(r), \overline{u}(r))$, $0 \le r \le 1$ which should satisfy the following requirements:

1. $\underline{u}(r)$ is a bounded left continuous nondecreasing function over [0, 1].
2. $\overline{u}(r)$ is a bounded left continuous nonincreasing function over [0, 1].
3. $\underline{u}(r) \le \overline{u}(r)$, $0 \le r \le 1$.

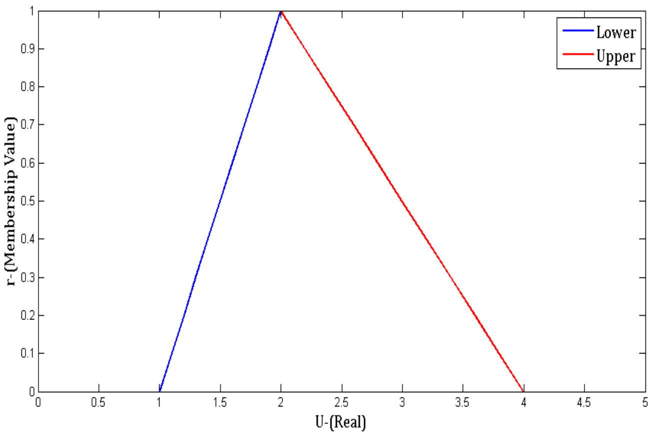

Fig. 1 A fuzzy number

For example, the fuzzy number (1 + r, 4 - 2r) is shown in Fig. 1. A crisp number α is simply represented by $\underline{u}(r) = \overline{u}(r) = \alpha$, $0 \le r \le 1$.

By appropriate definitions the fuzzy number space $\{\underline{u}(r), \underline{u}(r)\}$ becomes a convex cone E^1 which is then embedded isomorphically and isometrically into a Banach space.

Definition 1. The n × n linear system of equations

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= y_1, \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= y_2, \\ &\vdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n &= y_n \end{aligned} \quad (1)$$

Where the coefficient matrix $A = (a_{ij})$, $1 \le i,j \le n$ is a crisp $n \times n$ matrix and $y_i \in E^1, 1 \le i \le n$ is called a FSLEs.

A solution $(x_1, x_2, \dots, x_n)^t$ to (1) one should recall that for arbitrary fuzzy numbers $x = (\underline{x}, \overline{x})$, $y = (\underline{y}, \overline{y})$ and real number k,

a. $x = y$ *if and only if* $\underline{x}(r) = \underline{y}(r)$ *and* $\overline{x}(r) = \overline{y}(r)$

b. $x + y = \underline{x}(r) + \underline{y}(r), \overline{x}(r) + \overline{y}(r)$

c. $kx = \begin{cases} (k\underline{x}, k\overline{x}), & k \ge 0, \\ (k\overline{x}, k\underline{x}), & k < 0 \end{cases}$ (2)

Definition 2[12]. A fuzzy number vector $(x_1, x_2, \dots, x_n)^t$ given by $x_i = (\underline{x}(r), \overline{x}(r)), 1 \le i \le n, 0 \le r \le 1$, is called a solution of the FSLE if

$$min\left\{\sum_{j=1}^{n} a_{ij}u_j \,|\, u_j \in [x_j]_r\right\} = \underline{\sum_{j=1}^{n} a_{ij}x_j} = \sum_{j=1}^{n} \underline{a_{ij}x_j} =$$

(3)

$$max\left\{\sum_{j=1}^{n} a_{ij}u_j \,|\, u_j \in [x_j]_r\right\} = \overline{\sum_{j=1}^{n} a_{ij}x_j} = \sum_{j=1}^{n} \overline{a_{ij}x_j} =$$

Consider the *i*th equation of the system (1):

$a_{i1}(\underline{x}_1, \overline{x}_1) + \cdots + a_{ii}(\underline{x}_i, \overline{x}_i) + \cdots + a_{in}(\underline{x}_n, \overline{x}_n) = (\underline{y}_i, \overline{y}_i)$

We have

$$\underline{a_{i1}x_1} + \cdots + \underline{a_{ii}x_i} + \cdots + \underline{a_{in}x_n} = \underline{y}_i(r),$$

(4)

$$\overline{a_{i1}x_1} + \cdots + \overline{a_{ii}x_i} + \cdots + \overline{a_{in}x_n} = \overline{y}_i(r), \qquad 1 \le i \le$$

From (4) we have two crisp n× n linear systems for all *i* that there can be extended to a 2n × 2n crisp linear system as follows:

$$SX = Y \rightarrow \begin{bmatrix} S_1 \ge 0 & S_2 \le 0 \\ S_2 \le 0 & S_1 \ge 0 \end{bmatrix} \begin{bmatrix} \underline{X} \\ \overline{X} \end{bmatrix} = \begin{bmatrix} \underline{Y} \\ \overline{Y} \end{bmatrix} \quad (5)$$

Thus FSLE(1) is extended to a crisp (5) where $A=S_1+S_2$. Eq. (5) can be write as follows:

$$\begin{cases} S_1\underline{X} + S_2\overline{X} = \underline{Y} \\ S_2\underline{X} + S_1\overline{X} = \overline{Y} \end{cases}$$

or

$$\begin{cases} S_1\underline{X} - S_2\overline{X} = \underline{Y}, \\ -S_2\underline{X} + S_1\overline{X} = \overline{Y}. \end{cases} \quad (6)$$

Theorem 1. The matrix S is nonsingular if and only if the matrices $A=S_1+S_2$ and $S_1 - S_2$ are both nonsingular. See [21].

Definition 3 Let $X = \{(\underline{x}_i(r), \overline{x}_i(r)), 1 \le i \le n\}$ denote the unique solution of $SX=Y$. The fuzzy number vector $U = \{(\underline{u}_i(r), \overline{u}_i(r)), 1 \le i \le n\}$ defined by

$$\begin{aligned} \underline{u}_i(r) &= min\{\underline{x}_i(r), \overline{x}_i(r), \underline{x}_i(1)\} \\ \overline{u}_i(r) &= max\{\underline{x}_i(r), \overline{x}_i(r), \underline{x}_i(1)\}, \end{aligned} \quad (7)$$

Is called the fuzzy solution of $SX=Y$. If $(\underline{x}_i(r), \overline{x}_i(r))$, $1 \le i \le n$, are all triangular fuzzy numbers then $\underline{u}_i(r) = \underline{x}_i(i), \overline{u}_i(r) = \overline{x}_i(r), 1 \le i \le n$ and U is called a strong fuzzy solution. Otherwise, U is a weak fuzzy solution. See [12].

IV. Fuzzy Complex System of Linear Equations

A. *Preliminaries*

We define an arbitrary fuzzy complex number [23, 24] by using two fuzzy numbers that represent real and imaginary part of complex number as:

$\tilde{Z} = \tilde{a} + \tilde{b}i$, where $\tilde{a} = (\underline{a}(r), \overline{a}(r))$,

$\tilde{b} = (\underline{b}(r), \overline{b}(r))$ and $0 \le r \le 1$

therefore

$\underline{Z} = \underline{a}(r) + i\underline{b}(r)$

and

$\overline{Z} = \overline{a}(r) + i\overline{b}(r)$

Definition 4. The n × n linear system of complex equations

$$\begin{aligned} c_{11}z_1 + c_{12}z_2 + \cdots + c_{1n}z_n &= w_1, \\ c_{21}z_1 + c_{22}z_2 + \cdots + c_{2n}z_n &= w_2, \\ &\vdots \\ c_{n1}z_1 + c_{n2}z_2 + \cdots + c_{nn}z_n &= w_n \end{aligned} \qquad (8)$$

Where the coefficient matrix $C = (c_{ij}), 1 \le i, j \le n$ is a complex crisp $n \times n$ matrix and $w_i, 1 \le i \le n$ is complex fuzzy number, is called a FCSLEs.

We can represent this system as:

$$\sum_{j=1}^{n} c_{ij} z_j = w_i \ \ , for\ i = 1,2, \dots, n$$

In order to solving this system we decomposed complex numbers:

$$c_{ij} = a_{ij} + ib_{ij}$$
$$z_i = p_i + iq_i$$
$$w_i = u_i + iv_i$$

$$\sum_{j=1}^{n} (a_{ij} + ib_{ij})(p_j + iq_j) = u_i + iv_i\ ,$$
$$for\ i = 1,2, \dots, n$$

Therefore

$$\sum_{j=1}^{n} a_{ij}p_j - b_{ij}q_j + i\sum_{j=1}^{n} a_{ij}q_j + b_{ij}p_j = u_i + iv_i\ ,$$
$$for\ i = 1,2, \dots, n$$

Now we can separate real and imaginary parts:

$$\sum_{j=1}^{n} a_{ij}p_j - b_{ij}q_j = u_i\ , for\ i = 1,2, \dots, n$$
$$\sum_{j=1}^{n} b_{ij}p_j + a_{ij}q_j = v_i\ , for\ i = 1,2, \dots, n$$

For solving this system we can write:

$$V = [v_i]$$
$$U = [u_i]$$
$$A = [a_{ij}]$$
$$B = [b_{ij}]$$
$$P = [p_i]$$
$$Q = [q_i]\ , for\ i, j = 1,2, \dots, n$$
$$\begin{bmatrix} A & -B \\ B & A \end{bmatrix} \begin{bmatrix} P \\ Q \end{bmatrix} = \begin{bmatrix} U \\ V \end{bmatrix} \qquad (9)$$

As you see this system is a FSLE that can be solved by approach considered in section 3.

V. Circuit Analysis with Fuzzy Variables

A linear time invariant electric circuit with constant coefficients and fuzzy sources has a system of fuzzy linear equations that can be expressed in the following form,

$$\begin{aligned} a_{11}\tilde{x}_1 + a_{12}\tilde{x}_2 + \cdots + a_{1n}\tilde{x}_n &= \tilde{y}_1 \\ a_{21}\tilde{x}_1 + a_{22}\tilde{x}_2 + \cdots + a_{2n}\tilde{x}_n &= \tilde{y}_2 \\ &\vdots \\ a_{n1}\tilde{x}_1 + a_{n2}\tilde{x}_2 + \cdots + a_{nn}\tilde{x}_n &= \tilde{y}_n \end{aligned}$$

Where a_{ij} is a constant coefficient, $\tilde{y}_i$ is a fuzzy source and $\tilde{x}_i$ can be current or voltage in the circuit. Thus we can solve this system with approaches considered in sections 3 and 4.

A. *Examples*

In this section we are going to solve a circuit example. We will in fact consider a fuzzified version of examples from classical circuit analysis[21, 22].

A.1 Example 1

Consider a simple resistive circuit with fuzzy current and fuzzy source

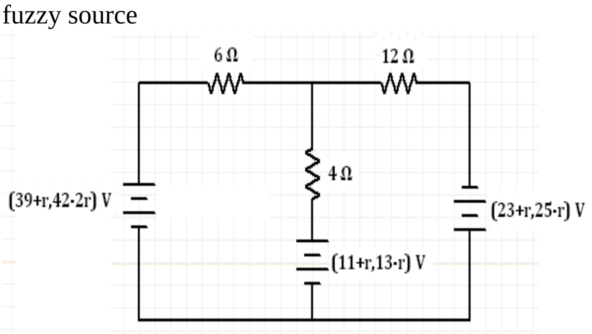

Fig. 2. A circuit with fuzzy current and fuzzy source

The system of equations for this circuit is:

$$10\tilde{I}_1 - 4\tilde{I}_2 = (39 + r, 42 - 2r) - (11 + r, 13 - r)$$
$$-4\tilde{I}_1 + 16\tilde{I}_2 = (11 + r, 13 - r) + (23 + r, 25 - r)$$

We can simplify system as:

$$\begin{cases} 10\tilde{I}_1 - 4\tilde{I}_2 = (26 + 2r, 31 - 3r) \\ -4\tilde{I}_1 + 16\tilde{I}_2 = (34 + 2r, 38 - 2r) \end{cases}$$

Therefore

$$S_1 = \begin{bmatrix} 10 & 0 \\ 0 & 16 \end{bmatrix}, S_2 = \begin{bmatrix} 0 & -4 \\ -4 & 0 \end{bmatrix}$$

And then

$$S = \begin{bmatrix} 10 & 0 & 0 & -4 \\ 0 & 16 & -4 & 0 \\ 0 & -4 & 10 & 0 \\ -4 & 0 & 0 & 16 \end{bmatrix}, \tilde{Y} = \begin{bmatrix} 26 + 2r \\ 34 + 2r \\ 31 - 3r \\ 38 - 2r \end{bmatrix}$$

$$S^{-1} = \begin{bmatrix} \frac{1}{9} & 0 & 0 & \frac{1}{36} \\ 0 & \frac{5}{72} & \frac{1}{36} & 0 \\ 0 & \frac{1}{36} & \frac{1}{9} & 0 \\ \frac{1}{36} & 0 & 0 & \frac{5}{72} \end{bmatrix}$$

$$\tilde{I} = \begin{bmatrix} \frac{71}{18} + \frac{1}{6}r \\ \frac{29}{9} + \frac{1}{18}r \\ \frac{79}{18} - \frac{5}{18}r \\ \frac{121}{36} - \frac{1}{12}r \end{bmatrix},$$

$$\tilde{I}_1 = \left(\frac{71}{18} + \frac{1}{6}r, \frac{79}{18} - \frac{5}{18}r\right),$$

$$\tilde{I}_2 = \left(\frac{29}{9} + \frac{1}{18}r, \frac{121}{36} - \frac{1}{12}r\right)$$

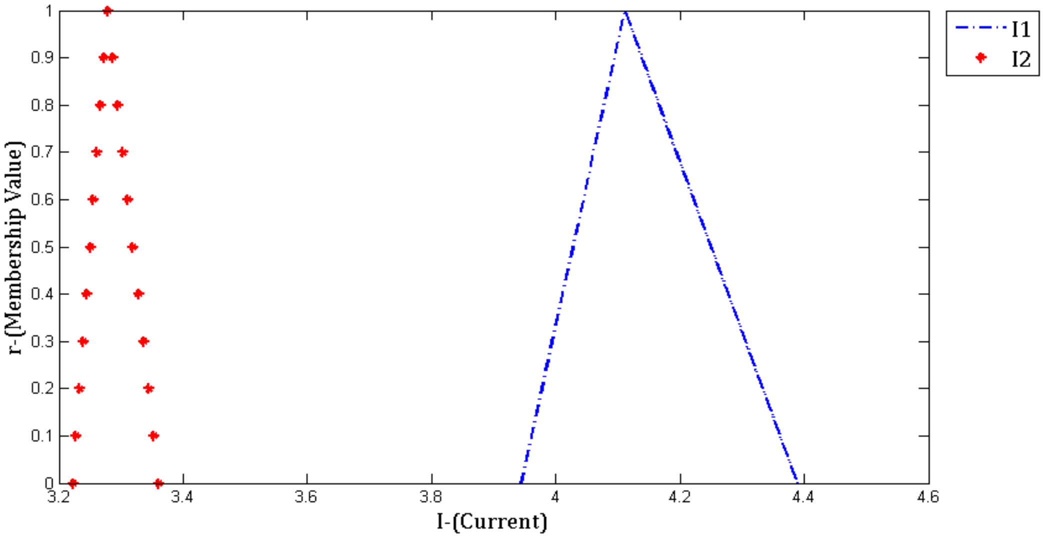

Fig. 3. The solution of the system from example 5.1.1

A.2 Example 2

Consider a simple RLC circuit with fuzzy current and fuzzy source in figure 4.
The system of equations for this circuit is:

$$\begin{cases} (10 - 7.5j)I_1 - (6 - 5j)I_2 = \\ (4 + r, 6 - r) + j(-1 + r, 1 - r) \\ \\ -(6 - 5j)I_1 + (16 + 3j)I_2 = \\ (-2 + r, -r) + j(-3 + r, -1 - r) \end{cases}$$

In matrix form, we obtain

$$I_1 = \tilde{x}_1 + i\tilde{y}_1$$
$$I_2 = \tilde{x}_2 + i\tilde{y}_2$$

$$\begin{bmatrix} 10 - 7.5i & -6 + 5i \\ -6 + 5i & 16 + 3i \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} (4 + r, 6 - r) + j(-1 + r, 1 - r) \\ (-2 + r, -r) + j(-3 + r, -1 - r) \end{bmatrix}.$$

In order to (9)

$$A = \begin{bmatrix} 10 & -6 \\ -6 & 16 \end{bmatrix}, B = \begin{bmatrix} -7.5 & 5 \\ 5 & 3 \end{bmatrix}$$
$$U = \begin{bmatrix} (4 + r, 6 - r) \\ (-2 + r, -r) \end{bmatrix}$$

$$V = \begin{bmatrix} (-1+r, 1-r) \\ (-3+r, -1-r) \end{bmatrix}$$

$$P = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \; Q = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

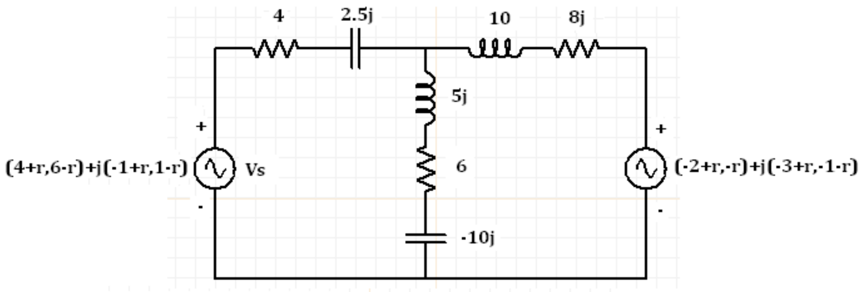

Fig. 4. A RLC circuit with fuzzy current and fuzzy sources

Therefore

$$\begin{bmatrix} 10 & -6 & 7.5 & -5 \\ -6 & 16 & -5 & -3 \\ -7.5 & 5 & 10 & -6 \\ 5 & 3 & -6 & 16 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} (4+r, 6-r) \\ (-2+r, -r) \\ (-1+r, 1-r) \\ (-3+r, -1-r) \end{bmatrix}$$

In order to 5

$$S_1 = \begin{bmatrix} 10 & 0 & 7.5 & 0 \\ 0 & 16 & 0 & 0 \\ 0 & 5 & 10 & 0 \\ 5 & 3 & 0 & 16 \end{bmatrix}$$

$$S_2 = \begin{bmatrix} 0 & -6 & 0 & -5 \\ -6 & 0 & -5 & -3 \\ -7.5 & 0 & 0 & -6 \\ 0 & 0 & -6 & 0 \end{bmatrix}$$

$$S = \begin{bmatrix} 10 & 0 & 7.5 & 0 & 0 & -6 & 0 & -5 \\ 0 & 16 & 0 & 0 & -6 & 0 & -5 & -3 \\ 0 & 5 & 10 & 0 & -7.5 & 0 & 0 & -6 \\ 5 & 3 & 0 & 16 & 0 & 0 & -6 & 0 \\ 0 & -6 & 0 & -5 & 10 & 0 & 7.5 & 0 \\ -6 & 0 & -5 & -3 & 0 & 16 & 0 & 0 \\ -7.5 & 0 & 0 & -6 & 0 & 5 & 10 & 0 \\ 0 & 0 & -6 & 0 & 5 & 3 & 0 & 16 \end{bmatrix}$$

$$Y = \begin{bmatrix} 4+r \\ -2+r \\ -1+r \\ -3+r \\ 6-r \\ -r \\ 1-r \\ -1-r \end{bmatrix}$$

After solving the above system we have

$$X = \begin{bmatrix} 0.3164 + 0.0378r \\ 0.0347 + 0.0307r \\ 0.0708 + 0.0378r \\ -0.2380 + 0.0307r \\ 0.3920 - 0.0378r \\ 0.0961 - 0.0307r \\ 0.1464 - 0.0378r \\ -0.1765 - 0.0307r \end{bmatrix}$$

Therefore

$$I_1 = (0.3164 + 0.0378r, 0.3920 - 0.0378r) + i(0.0708 + 0.0378r, 0.1464 - 0.0378r)$$

$$I_2 = (0.0347 + 0.0307r, 0.0961 - 0.0307r) + i(-0.2380 + 0.0307r, -0.1765 - 0.0307r)$$

The crisp solution for circuit represented in figure 4 is:

$I_1 = 0.3542 + 0.1086i$
$I_2 = 0.0654 - 0.2072i$
That equals with the fuzzy solution with r=1.

B. Sensitivity check

If we change the V_s source from 5 to 5.1 the solution will be
$I_1 = 0.3629 + 0.1119i$
$I_2 = 0.0693 - 0.2095i$

That is within the α-cut with α=0.75 in fuzzy solution and is an acceptable solution for this system. This simple experience shows how fuzzification of the system can help maintain the tolerance of the system. Therefore, fuzzy current obtained from solution of FCSLE can easily cover some change in value of elements, this property can be of great use in designing procedure.

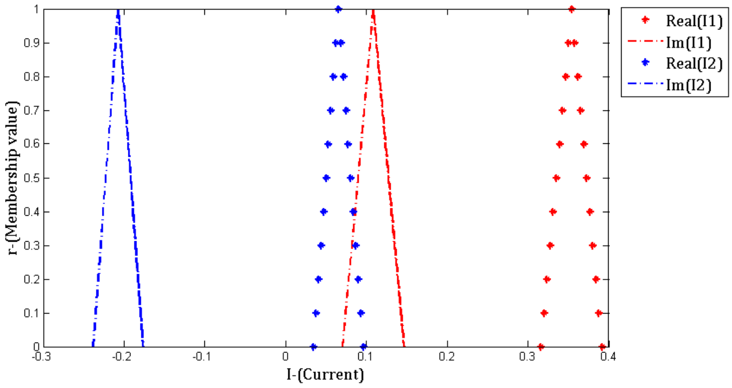

Fig. 5. The solution of the system from example 5.1.2

VI. CONCLUSION

Actual simulation of real world is the major problem in many fields of engineering applications. In circuit design, simulation of real current and voltage is major problem in circuit analysis and design. We tried to present a fuzzy based approach for simulation of voltage and current sources and fuzzy current as well as voltage in circuit equations. Fuzzy complex system of linear equations was derived for this purpose. The proposed scheme could cover changes in circuit elements (value of element).

REFERENCES

[1] L. W. Nagel, "Spice2: A Computer Program to Simulate Semiconductor Circuits," Memo. no. ERL-M520, Electronics Research Laboratory, UC, Berkeley, May 1975.

[2] C. Chao and L. Milor. "Performance Modeling of Analog Circuits Using Additive Regression Splines" lEEE 1994 Custom Integrated Circuit Conference, pages 301-304,1994.

[3] J. Shao and R. Hajani. "Feasibility Region Modeling of Analog Circuits for Hierarchical Circuit Design" IEEE Midwest Symposium on Circuits and Systems, 1994.

[4] A. Torralba, J. ChBvez, and L. Franquelo. "Circuit Performance Modeling by Means of Fuzzy Logic" IEEE Trans. on Computer-Aided Design of Integrated Circuits and Systems, 15(11):1391-1398, Nov. 1996.

[5] E. S. Kuh, I. N. Hajj, "Nonlinear Circuit Theory: Resistive Networks," Proceedings of the IEEE. Vol.59, no.3, Mar. 1971.

[6] M. K. Oksal, S. Herdem, "Analysis of Nonlinear Circuits by Using Differential Taylor Transform," Computers and Electrical Engineering 28 (2002) 513–525.

[7] Bedrosian D, Vlach J. "Time domain analysis of network with internally controlled switches" IEEE Trans Circuits Sys-1 1992;39:192.

[8] T. Mei, J. Roychowdhury, T. S. Coffey, S. A. Hutchinson, D. M. Day, "Robust, Stable Time Domain Methods for Solving MPDEs of Fast/Slow Systems," Design Automation Conference, 41st Conference on (DAC'04), pp. 848-853, June 2004.

[9] H. Sadoghi Yazdi, M. Pakdaman, S. Effati, "Fuzzy Circuit Analysis," Int. J. of Applied Engineering Research, Vol.3, Issue 8, 2008.

[10] M. Friedman, M. Ma, A. Kandel, "Fuzzy linear systems" Fuzzy Sets and Systems 96 (1998) 201–209.

[11] T. Allahviranloo, "Numerical methods for fuzzy system of linear equations," Appl. Math. Comput. 155 (2004) 493–502.

[12] T. Allahviranloo, "The Adomian decomposition method for fuzzy system of linear equations," Appl. Math. Comput. 163 (2005) 553–563.

[13] T. Allahviranloo, "Successive over relaxation iterative method for fuzzy system of linear equations," Appl. Math. Comput. 162 (2005) 189–196.

[14] M. Dehghan, B. Hashemi, "Iterative solution of fuzzy linear systems," Applied Mathematics and Computation, in press, doi:10.1016/j.amc.2005.07.033.

[15] M. Ma, M. Friedman, A. Kandel, "Duality in fuzzy linear systems," Fuzzy Sets and Systems 109 (2000) 55–58.

[16] S. Muzzioli, H. Reynaerts, "Fuzzy linear systems of the form A1x + b1=A2x + b2," Fuzzy Sets and Systems 157 (2006) 939 – 951

[17] J.J. Buckley, Y. Qu, "Solving systems of linear fuzzy equations," Fuzzy Sets and Systems 43 (1991) 33–43.

[18] S. Abbasbandy a,b, A. Jafarian a,c, "Steepest descent method for system of fuzzy linear equations," Applied Mathematics and Computation 175 (2006) 823–833

[19] C. Wu, M. Ma, "Embedding problem of fuzzy number space: Part I," Fuzzy Sets and Systems 44 (1991) 33–38.

[20] M. Dehghan , B. Hashemi, M. Ghatee, "Computational methods for solving fully fuzzy linear systems," Applied Mathematics and Computation 179 (2006) 328–343

[21] Attia, John Okyere. Matlab Fundamentals. Electronics and Circuit Analysis using MATLAB, CRC Press LLC 1999, ISBN 0-8493-1176-4

[22] John O'Malley, Theory and problems of Basic Circuit Analysis, Second Edition. ISBN 0-07-047824-4.

[23] J. J. Buckley, "Fuzzy Complex Numbers," Proceeding of ISFK, Guangzhou, China, 1987, 697-700.

[24] J. J. Buckley, "Fuzzy Complex Numbers," Fuzzy Sets and Systems, vol. 33, pp. 333-345, 1989.

Taher Rahgooy was born in Ilam, Iran, in 1982. He received the B.S degree in Computer engineering from University of Isfahan, Iran, in 2006. He is now a M.S student in Computer Engineering in Ferdowsi University of Mashhad, Iran. His research interests include soft computing and knowledge discovery.
E-mail:Taher.Rahgooy@gmail.com

Hadi Sadoghi Yazdi received the B.S. degree in electrical engineering from Ferdowsi Mashhad University of Iran in 1994, and then he received the M.S. and PhD degrees in electrical engineering from Tarbiat Modarres University of Tehran, Iran, in 1996 and 2005 respectively. He works in Engineering Department as an assistant professor at Ferdowsi University of Mashhad. His research interests include adaptive filtering, image and video processing, and optimization in signal processing. He has more than 140 journal and conference publications in subject of his interests.
E-mail:sadoghi@sttu.ac.ir

Reza Monsefi received the B.S. degree in electrical engineering from Manchester University from UK, MSC and PhD from Salford University UK in data communication and supervisory control. He has been with Ferdowsi university of Mashhad from FUM since 1991.