System Identification and H_∞ Observer Design for TRMS

Vidya S. Rao, Milind Mukerji, V. I. George, Surekha Kamath, and C. Shreesha

Abstract—A dynamic model for the two-degree-of-freedom Twin Rotor MIMO System (TRMS) is extracted using a black-box system identification technique. Its behaviour in certain aspects resembles that of a helicopter, with a significant cross coupling between longitudinal and lateral directional motions. Hence, it is an interesting identification and control problem. Using the extracted model an H_∞ observer is designed which will estimate the states of the system in presence of worst case noise assumed to be impact on the system.

Index Terms—Black box system identification, H_∞ observer, TRMS, ARMAX.

I. INTRODUCTION

The Twin Rotor MIMO System (TRMS) resembles the helicopter system in behaviour with significant cross coupling between the longitudinal and lateral axes. The difference between the TRMS and helicopter system is just that while the helicopter system varies the angle of the rotor blades to produce more or less force, the TRMS varies the speed of the D.C. motor.

As modeling of the TRMS is difficult due to non linearities and cross coupling system identification method is used to get a better model of the system.

System identification uses statistical methods to build mathematical models of dynamical systems from measured data. The system identification toolbox of MATLAB is a good way of estimating models for systems that are difficult to model. System Identification Toolbox constructs mathematical models of dynamic systems from measured input-output data. Black Box identification doesn't assume anything about the system and thus gives a good estimate of the system's characteristics.

As in the TRMS setup, the pitch angle and yaw angle can be measured and the other states are not available for feedback. So an observer is needed in order to estimate the intermediate states.

The low frequency inputs of range [0-1Hz] is selected and used to identify the system model of the TRMS and then it is reduced to get a 9th order model.

II. EXPERIMENTAL SETUP

The TRMS is shown in Fig. 1. It has a main rotor and a tail rotor for varying the pitch angle and yaw angle respectively. The two rotors are placed on the opposite sides with a counter balance in between. The whole unit is attached to a support to safely perform control experiments.

Apart from the mechanical unit, the electrical unit placed under the support allows easy transfer of signals from the sensors to PC and control signal via I/O card. The bound for control signal is (-2.5V to + 2.5 V) [1].

Fig. 1. The twin rotor MIMO system.

III. SYSTEM IDENTIFICATION

Although model based controllers are desirable, detailed models are expensive and difficult to arrive at from first principles and they generally cannot explain the noise and thus instead of deterministic models, probabilistic models are more desirable. Although the models developed from statistical methods are an approximation of the real model, it is good enough for control purposes. This involves collecting a lot of plant data and modelling of noise processes. System Identification process is shown in Fig. 2.

Fig. 2. Process of system identification.

Once appropriate measurements are made, the plant model is obtained. It involves two steps

- 1) Identification of a model structure
- 2) Estimation of parameter values relating to this model structure

In this paper ARMAX (Auto Regressive Moving Average Exogenous) model is used to approximate the TRMS system. In this model the current output is a function of previous outputs (auto regressive part, $A(q)y(t)$), past inputs (exogenous part, $B(q)u(t)$) and current and previous noise terms (moving average part, $C(q)e(t)$) [2],[3].

ARMAX models are of the form as given in "(1)"

$$A(q)y(t) = B(q)u(t) + C(q)e(t) \quad (1)$$

where q is the shift operator.

Manuscript submitted on March 7, 2013; revised June 30, 2013.

The authors are with Instrumentation & Control Engineering Dept., MIT, Manipal (e-mail: raovid@yahoo.co.in, vig_rect@yahoo.com, Surekakamath2k4@yahoo.com, shreesha.c@manipal.edu).

$$qu(t) = u(t+1) \text{ and } q^{-1}u(t) = u(t-1).$$

IV. EXPERIMENTATION

To estimate a model of the TRMS we give mixed sine waves of varying frequencies between 0-1Hz according to [1] to the system and then record both the input and output and give them as input to the MATLAB System identification toolbox which estimates a model. Here we choose the best fit model for each of the four pairs of input – outputs. We use the ARMAX(Auto Regressive Moving Average Extra) model to get an initial estimate[4]-[6]. Following are the best fit models:

- 1) Main yaw – amx101011
- 2) Main pitch – amx10023
- 3) Cross yaw - amx10023
- 4) Cross pitch - amx 10333

Fig. 3. TRMS block diagram.

V. H_∞ OBSERVER

A. Observer Design

Fig. 4. Observer design with state feedback.

An observer is used to estimate states that are not available for measurement or feedback. The observer basically works on minimizing $(y - \hat{y})$, which then leads to a good estimation of the states. The performance of the observer (4) depends on the value of observer gain, K which can be a static gain or a gain scheduled parameter.

B. H_∞ Observer

A filter is used to separate noise from actual measurements and thus estimate the correct value of the measured variable. A static filter is used to filter out low frequency or high frequency noise but the actual process noise may not be as well defined. The Kalman filter makes an assumption that all noise is white which may not be true

for all process noise which leads to bad performance and sometimes instability when the process noise is significant and non white. The H_∞ filter makes no such assumptions about the noise. It is designed for keeping the system stable for even the worst case noise. Also accurate system models are not as readily available in the industries, thus making kalman filter implementation difficult. The H_∞ estimator minimizes the worst case estimation error [7].

VI. H_∞ OBSERVER DESIGN

After obtaining the system model, it is converted into state space. System matrix, A , from the 9th order approximated model and then it is used to realize a full order H_∞ observer whose gain is decided by the H_∞ filter so that the predicted output $\hat{y}$ is as close to actual output y as possible in spite of measurement noises and other noises that corrupt the final output measurement. From this observer all nine states are estimated. While designing the H_∞ filter, $P(0)$ and $x(0)$ are assumed to be zero, Q, R, S as identity matrices.

The game theory approach is used to design H_∞ filter. The goal of designing an H_∞ filter is to find the correct observer gain K which minimizes the difference between the predicted output and the true output. Here, by varying the observer gain the H_∞ filter decides which output to place more emphasis on. Its task is to place less emphasis on noisy measurements and more emphasis on actual measurements. We can also design a steady state filter which assumes that the noise is constant but in this project we design a dynamic real time filter which changes the gain of the observer as the noise changes. [7]

Let us consider a continuous time linear system as in “(2)”

$$\begin{aligned} \dot{x} &= Ax + Bu + w \\ y &= Cx + v \\ z &= Lx \end{aligned} \quad (2)$$

where L is the user-defined matrix and z is the vector that we want to estimate. The estimate of z is denoted by $\hat{z}$ and the estimate of state at time 0 is $\hat{x}(0)$. The vectors w and v are disturbances with unknown statistics, they may not even be zero mean. Y is the system output and x is the state matrix. A, B, C are the system matrices of the system. The cost function used is given in “(3)”

$$J = \frac{\int_0^T \|z - \hat{z}\|^2 dt}{\|x(0) - \hat{x}(0)\|^2 + \int_0^T (\|w\|^2 + \|v\|^2) dt} \quad (3)$$

where P_0, Q, R, S are positive definite matrices chosen by the designer based on a specific problem. Our goal is to find an estimator such that

$$J < \frac{1}{\theta}$$

The estimator that solves this problem is given by $P(0) = P_0$

$$\dot{P} = AP + PA^T + Q - KCP + \theta PL^T SLP$$

$$K = PC^T R^{-1}(5)\hat{x} = A\hat{x} + Bu + K(y - C\hat{x})$$

$$\hat{z} = L\hat{x}$$

where K is the observer gain

This is the filter which is realized using MATLAB for the TRMS.

The simulation result for two of the states is shown in Fig. (9).

For choosing the Q matrix, we adopt a trial and error method where we simulate the response of the states and increase the value of the diagonal element of Q so that it affects the value of K more or less. The value of R is also selected similarly and it affects all states equally. So an increase in the value of R will change the response of all the states. We keep S at 1 because we achieve good response by varying Q and R . If Q is high and R is low, the observer performs well with plant uncertainty but is affected by noise. When Q is low and R is high, observer is less susceptible to noise but is affected by plant uncertainty. So there must be a compromise between Q and R according to the specific situation.

VII. SIMULATION RESULTS

Table I. shows the percentage fit of the data with the various models. The highest percentage fit is used as the correct model.

TABLE I: COMPARISON OF DIFFERENT MODELS OF TRMS

Main Pitch	% fit	Main Yaw	% fit	Cross pitch	% fit	Cross Yaw	% fit
Amx 6221	38.2	Arx 10105	57.04	Amx 10333	62	Amx 101023	70.23
Amx 4141	31.3	Amx 4221	39.55	N4s2	45.66	Amx 6234	55.23
Amx 2131	29.5	Amx 2113	33.21	Amx 6111	49.12	Amx 4112	45.44
Amx 10023	46.2	Amx 101011	44.22	Amx 4212	37.22		

A. TRMS Validation Results

Fig. 5. Yaw model validation.

Fig. 6. Pitch model validation.

Fig. 7. Cross pitch to yaw model validation.

Fig. 8. Cross yaw to pitch model validation.

These models are reduced and converted into continuous models. The transfer functions of TRMS are found as below.

$$1) \text{ Main pitch: } \frac{-1.9 \times 10^{-6} s^3 + 0.000169 s^2 + 0.015 s + 1.274}{s^3 + 1.193 s^2 + 4.283 s + 3.514}$$

$$2) \text{ Main yaw: } \frac{0.001922 s^2 - 0.05065 s + 0.2463}{s^2 + 0.3815 s + 0.3534}$$

$$3) \text{ Cross pitch: } \frac{-0.01031 s^2 + 0.02719 s + 0.6054}{s^2 + 1.1676 s + 1.161}$$

$$4) \text{ Cross yaw: } \frac{0.04858 s + 0.2051}{s^2 + 0.92035 s + 3.152}$$

A comparison of the step response of the models found by system identification and the real time step response is shown in Fig. 9 to Fig. 12. (BLUE – Real time unit step response, GREEN – Identified model unit step response).

Fig. 9. Step response comparison for main pitch.

Fig.10. Step response comparison for main yaw.

Fig. 11. Step response comparison for cross pitch to yaw.

Fig. 12. Step response comparison for cross yaw to pitch.

The State space model of identified TRMS model is given in “(6)”.

$$\begin{aligned}
 A &= \begin{bmatrix} -0.38 & -0.7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.92 & -1.57 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1.67 & -1.16 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1.19 & -2.14 & -1.75 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \\
 B &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0.25 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \\
 C &= \begin{bmatrix} -0.05 & 0.49120 & 0 & -0.001 & 0.6174 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.0002 & 0.00750 & 0.637 & 0 \end{bmatrix} \\
 D &= \begin{bmatrix} 0.0019 & -0.0103 \\ 0 & 0 \end{bmatrix}
 \end{aligned}
 \tag{6}$$

B. H_{∞} Observer Simulation Results

Fig. 13. Simulation of the first state related to pitch for ramp input.

Blue – Noisy state.
Red – Actual state.
Green – Estimated state.

Fig. 14. Simulation of first state related to Yaw for ramp input.

Above were the simulation results for an observer designed to work for a high level of noise. Q is low, R is high. Table II. shows this effect in terms of actual Q and R values. Noise is given in the range of 0.1 to 1 in magnitude with reference being 1 and the plant uncertainty varying from 0.9 to $0.99 \times A$ to test the system.

 TABLE II: EFFECT OF DIFFERENT Q AND R VALUES ON PERFORMANCE

Q	R	First row of K	Performance for noise	Performance for Plant uncertainty
diag[1,20,1,0,10,30,10,10,30,20]	diag[10,0,100]	[-0.0916 0]	Can handle medium amount of noise.	Can handle small amounts of plant uncertainty
diag[1,20,1,0,10,30,10,10,30,20]	diag[1,1]	[-0.4880 0]	Can handle very low amount of noise.	Can handle large amount of plant uncertainty
diag[2,1,30,20,20,2,2,5,2]	diag[20,0,200]	[-0.0014 0]	Can handle very high amount of noise.	Can handle very small amount of plant uncertainty

The results for the case when Q is high and R is low so that observer works for plant uncertainty shown in Fig. 15. In Fig. 16, A matrix is changed to $0.9 \times A$ with the new Q and R values.

Fig. 15. Simulation of first state related to Yaw for step input.

Fig. 16. Simulation of the first state related to pitch for step input.

VIII. CONCLUSION

In this paper a model for the Twin Rotor MIMO System is successfully identified. Then reduced model of order 9 is obtained for TRMS which is used in designing an H_∞ observer for the TRMS. It was observed from the simulation results that H_∞ observer designed gives good response in the presence of high level of noise input. If noise is not high, a higher value of observer gain gives good result which takes care of plant uncertainty.

REFERENCES

- [1] *Twin Rotor MIMO System Manual*, Feedback Instruments Ltd., U.K, 33-949S, 2002.
- [2] L. Ljung, *System identification, Theory for the user*, University of Linköping Sweden, Prentice Hall publishers, 1987.
- [3] M. Ahmad, A. J. Chipperfield, and M. O. Tokhi, "Dynamic Modeling and Control of a 2 DOF Twin Rotor," American control conference, vol. 3, pp. 32-36, 2000.
- [4] M. Ahmad, A. J. Chipperfield, and M. O. Tokhi, "Dynamic Modeling and Optimal Control of Twin rotor MIMO System," *IEEE proc.*, pp. 391-398, 2000.
- [5] S. M. Ahmed, A. J. Chipperfield, M. O. Tokhi, Rahideh, and M. H. Shaheed, "Dynamic modeling of a twin-rotor multiple input-multiple output system," in *Proc. Instn Mech Engrs*, vol. 216, Part I: J. Systems and Control Engineering, pp. 477-496, 2002.
- [6] A. Rahideh and M. H. Shaheed, "Mathematical dynamic modeling of a twin-rotor multiple input-multiple output system," in *Proc. IMechE*, vol. 221, Part I: J. Systems and Control Engineering, pp. 89-101, 2007.
- [7] D. Simon, *Optimal state estimation*, John Wiley Publication, 2006.

Vidya S. Rao was born in Manipal. She obtained B.E - Electrical & Electronics, Karnataka Regional Engineering College, Surathkal, Karnataka, India, 1996 and M-tech- Control Systems, Manipal Institute of Technology, Manipal, Karnataka, India, 2007. She is also a member of ACDOS. Pursuing Phd in Manipal University. Area of interest is H_∞ observer design and H_∞ controller design. This paper is the part of her research work.

She has nine years of teaching experience. Currently working as an Assistant Professor, Instrumentation & Control Engineering Dept., MIT, Manipal, Karnataka, India.

Milind Mukerji was born in Manipal. He obtained B.E Final Semester, Dept. Instrumentation and Control Engineering, Manipal Institute of Technology, Manipal. He is currently pursuing a B.E. in Instrumentation and control engineering.

Mr. Mukerji is interested in the fields of Control systems, Process control, Robotics and embedded systems.

VI George was born in Jaipur. He obtained B.E - Electrical and Power Systems, Manipal Institute of Technology, Karnataka, India, 1983 and M-tech- Instrumentation and Control Engineering, NIT, Calicut, 1987. He has received Phd - NIT Thrichy, Area of interest is Control System and aero space.

He has twenty seven years of teaching experience and eleven years of research experience. Currently working as director of Electrical Engineering, Jaipur, MU. He was the Head of the department and a Department Curriculum Committee Member.

Dr. George has won the Manipal university incentive award two times, IE award, rashtriyagaurav award, 2010, Vikram award, 2010.

Surekha Kamath was born in Manipal. She obtained B.E. in BDT College of Engineering, Davanagere, Karnataka, India and M-Tech - Biomedical Engineering, MIT, Manipal as well as Phd - Manipal University, Manipal.

She is now an associate professor in department of instrumentation and control Engineering, MIT, Manipal. Area of interest is Biomedical Engineering and Robust Control. She is a member of Institution of Engineers and BMESI. Dr. Kamath has published many journal and conference papers.

Shreesha Chokkadi was born in Manipal. He obtained B.E. E&E in BDT Engineering College, Davanagere, Karnataka, India, 1988, M Econtrol systems (Electrical), Walchand College of Engineering Sangli, 1992 as well as Ph.D. IIT Bombay, 2002. He is currently working as professor and Head of the Department of Instrumentation and Control Engineering, Manipal Institute of Technology, Manipal. Dr. Chokkadi's areas of interest in teaching are Linear and Nonlinear Controls, Modern control Theory, Optimal control theory, Network theory, Digital Signal Processing. Dr. Chokkadi is a member of FIE, M ISTE, M ISLE.