The T-Mixture Model Approach for Radar HRRP Target Recognition

Caiyun Wang and Jiling Xie

Abstract—This paper addresses the problem of automatic radar target recognition using high-resolution range profile (HRRP). We develop a t-mixture model (TMM) to model radar echoes from each target as the t-distribution forms. Estimation of the model parameters is achieved using a modified expectation-maximization (EM), i.e., fuzzy EM (FEM) algorithm. Numerical simulation results show that the proposed approach can achieve good performance of robustness and higher recognition rate. It is helpful for real-time identification and the engineering implements of automatic target recognition using HRRP.

Index Terms—High-resolution range profile (HRRP), radar automatic target recognition (RATR), t-mixture model (TMM), statistical model, expectation-maximization (EM).

I. INTRODUCTION

Radar High Resolution Range Profiles (HRRP) can be seen as the one-dimensional projection result of the electromagnetic scattering characteristics of the target as a function of the range along the radar Line-of-sight (LOS). It reflects the geometrical shape and structural characteristics of the numbers of strong scattering centers the relative position of scattering centers and the relative magnitude of scattering strength included in the target. Recently HRRP has been used as major signatures for radar automatic target recognition (RATR) [1]-[13].

From the view of signal representation, HRRP recognition methods can be roughly classified into two categories: methods directly using range profiles as classification features [1]; methods based on target features extracted from range profiles [2], [3]. References [4], [5] studied some time-shift invariant features of HRRP; [6], [7] discussed the variation of recognition performance with signal bandwidth; some literature showed that statistical recognition is an efficient method for RATR [8]-[11]. Beckman [9] proposed a model based on beta distribution, Gaussian model [8], Gamma model [10] and the two-distribution compounded model [11] was also proposed for HRRP samples. But the performance of existing statistical models is unsatisfactory in target identification. Beta model [9] is sensitive to target aspect. Though the use of Gamma component distribution [10] arises from physical consideration, this model has computational reasons. The two-distribution compounded

model [11] comprising Gamma distribution and Gaussian mixture distribution is a complicated one.

Multivariate Gaussian components for mixtures are ubiquitously focused on due to their analytical tractability, asymptotic properties and computational convenience. However, as stated by Peel and Shoham [12]-[13], the mixtures of Gaussians lack robustness in the presence of the observations that are atypical of the components, and therefore the fitting of mixtures of t-distributions is usually adopted alternatively as a standard choice. The t-distribution has a wider tail than the Gaussian distribution, and is immune from atypical observations of a component by giving reduced weight in the calculation of its parameters.

An Expectation-Maximization (EM) algorithm for mixtures of multivariate t-distributions algorithm gives a general solution for parameter estimation [14]. Unfortunately, traditional EM algorithm leads to poor parameter estimation when a single run of EM is used, for suboptimal solutions. Fuzzy C-Means method has a tendency to recommend solutions that equalize cluster populations. Therefore, we use the fuzzy expectation-maximization (FEM) algorithm to estimate the t-distribution mixture model parameters.

In this paper, we consider multivariate t-mixture model (TMM) for target HRRP. Parameter estimation of t-mixture distribution is achieved by using the FEM algorithm. In the recognition experiment based on simulated data, the proposed statistical model not only has better recognition performance but also is more robust to noises than other methods.

The rest of the paper is organized as follows. In Section II, the TMM statistical model approach to HRRP-based statistical recognition is proposed. Section III introduces the modified EM algorithm, i.e., FEM used to estimate the parameters of the mixture model. The results from an application to radar HRRP target recognition are given in Section IV, and compared with GMM techniques. Finally, we present our conclusions in Section V.

II. STATISTICAL MODEL FOR TARGET HRRP

A. Introduction to Mixture Models

According to Bayes theorem, we have $P(c|\mathbf{x}) \propto p(\mathbf{x}|c)P(c)$, where $p(\mathbf{x}|c)$ is the class-conditional probability density and $P(c)$ is the prior class probability. Thus, estimation of the posterior probability is turned into estimation of the class-conditional density. For an HRRP including many range cells, nonparametric methods of density estimation, e.g., Parzen window method, are impractical due to the huge storage requirement. An

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alternative approach is to trade flexibility for robustness and to use some simple parametric form for target probability density. Mixture models [15] provide a convenient semi-parametric method for density estimation, and have been used in some form or another in numerous areas, including target tracking, speech recognition.

The general form of a mixture model is given by the following distribution

$$p(\mathbf{x} | \Theta) = \sum_{m=1}^M \pi_m p_m(\mathbf{x} | \theta_m) \quad (1)$$

with

$$\sum_{m=1}^M \pi_m = 1, (0 \leq \pi_m \leq 1) \quad (2)$$

where $\Theta = \{\pi_m, \theta_m\}$, ($m=1,2,\dots,M$) denotes the parameter set. π_m are the mixing probabilities, and $p_m(\mathbf{x} | \theta_m)$ the parameterized component densities.

Given parameterized forms for the component densities, there are many techniques for estimating the parameters θ_m . The most common is the Expectation-Maximization (EM) algorithm, which estimates the maximum likelihood parameters. EM algorithm has attracted a great deal of interest over the past few years in a wide range of application involving tasks with incomplete data sets [16]-[17].

B. T-Mixture Component of Target HRRP

It is a great challenge in HRRP-based statistical recognition to give the accurate description of HRRPs' statistical characteristics. Typically, for continuous densities, the components are Gaussian and Gamma. As mentioned in the "Introduction" section, it is more reasonable to view the atypical observations from a t-mixture model (TMM). In the TMM, it is assumed that the data are from several components with distinct t-distributions. In this work we use t-distributions.

Suppose $\{x_1, x_2, \dots, x_N\}$ are D -dimensional random observations of size k and are generated independently from a mixture of M underlying populations with unknown proportion

$$p(\mathbf{x}_k | \Psi) = \sum_{m=1}^M \pi_m p_D(\mathbf{x}_k; \mu_m, \Sigma_m, \nu_m) \quad (3)$$

with

$$p_D(\mathbf{x}_k; \mu_m, \Sigma_m, \nu_m) = \frac{\Gamma\left(\frac{\nu_m + D}{2}\right) |\Sigma|^{-1/2}}{(\pi \nu_m)^{D/2} \Gamma\left(\frac{\nu_m}{2}\right)} \times \left(1 + \frac{\delta(\mathbf{x}_k, \mu_m; \Sigma_m)}{\nu_m}\right)^{-(\nu_m + D)/2} \quad (4)$$

$$\Psi = \{\mu_m, \Sigma_m, \nu_m\}, (m = 1, 2, \dots, M) \quad (5)$$

where $\Psi = \{\pi_m, \Psi_m\}$ denotes the set of model parameters and M the mixture number of densities in the model. In (4), each mixture component $p_D(x_k; \mu_m, \Sigma_m, \nu_m)$ is the conditional

probability density function (pdf.) of x_k with the parameter Ψ_m , which can be interpreted as the pdf. of those data points that form a corresponding cluster, say Cluster m , and π_m is the proportion of data points that belong to Cluster m . In (5), $\Psi_m = \{\mu_m, \Sigma_m, \nu_m\}$ denotes the density of the t-distribution with mean μ_m , covariance matrix Σ_m , and degree of freedom ν_m .

As we know, if $\nu > 1$, μ is the mean of the population, and if $\nu > 2$, Σ is the covariance matrix. The case $\nu \rightarrow +\infty$ corresponds to a multivariate Gaussian distribution with mean μ and covariance matrix Σ , and when $\nu = 1$, this becomes multivariate Cauchy distribution. So the value of ν can tune the distribution, and t-distribution is more robust than Gaussian or Gamma distributions.

As aforementioned, a t-mixture model can describe the statistical characteristics of HRRPs more accurately than Gaussian model. This paper proposed that echoes of different types of cells are modeled as the t-distribution form.

III. PARAMETER ESTIMATION

When a statistical model is available, the classifier training problem is turned into a parameter estimation problem.

There are three sets of parameters to estimate: the values of π_m , the component of the vectors Ψ_m and the value of M . As we know, the more mixture components the t mixture distribution has, the more precise the estimated pdf. will be. However, too complex data representation model may result in overfitting and cannot guarantee good generalization performance. The estimation of M is rather difficult, even though some techniques have been proposed. In this paper, we assume that M is known as four.

A. EM Algorithm

In the EM framework, the data are viewed as incomplete, and the complete version is given by

$$\mathbf{X}_C = \{\mathbf{X}, \mathbf{Z}_1, \dots, \mathbf{Z}_N\} \quad (6)$$

where $\mathbf{X} = \{x_1, x_2, \dots, x_N\}$ denotes the observed data set, and $\mathbf{Z}_k = (z_{k1}, z_{k2}, \dots, z_{kM})^T$ is the indicator variables $z_{km} = 1$ if x_k come from component m , zero otherwise, and $z_k, k=1,2,\dots,N$ are independently and identically distributed according to a multinomial distribution taking M mutually exclusive values with probability $\{\pi_1, \pi_2, \dots, \pi_M\}$. Regarding the indicator variables as missing data, a loglikelihood function can be written as

$$\mathcal{L}(\Psi | \mathbf{X}_C) = \sum_{k=1}^N \sum_{m=1}^M z_{km} [\ln p_D(\mathbf{x}_k; \mu_m, \Sigma_m, \nu_m) + \ln \pi_m] \quad (7)$$

We notice that the multivariate t-distribution can be viewed as a weighted average multivariate Gaussian distribution with the weight Gamma distribution. Therefore, we introduce another set of "missing data" W_k , when $z_{km} = 1$, $x_k | (w_k, z_{km} = 1)$ is independently and identically distributed (IID), and $w_k | (z_{km} = 1) \sim G(\nu_m/2, \nu_m/2)$, ($k=1,2,\dots,N$).

Now the “complete data” set can be rewritten as corresponding loglikelihood function is

$$\mathbf{X}'_C = \{\mathbf{X}, Z_1, \dots, Z_N, \mathbf{W}_1, \dots, \mathbf{W}_N\} \quad (8)$$

$$\begin{aligned} \mathcal{L}(\Psi | \mathbf{X}'_C) = & \sum_{k=1}^N \sum_{m=1}^M z_{km} [\ln p(\mathbf{x}_k | \mathbf{w}_k, z_{km} = 1; \Psi) \\ & + \ln p(\mathbf{w}_k | z_{km} = 1; \Psi) + \ln p(z_{km} = 1; \Psi)] \end{aligned} \quad (9)$$

As shown by Peel [15] et al., the ML estimate of Ψ can be achieved by iterating the following two steps

- E-Step: Given an input the $\mathbf{x}_k$, we fix $\Psi^{(t)}$ and cumulate $z_{km}^{(t)}$, $\mathbf{W}_k^{(t)}$ in (10), (11) respectively.

$$\hat{z}_{km}^{(t)} = \frac{p(\mathbf{x}_k | \Psi_m^{(t)}) \pi_m^{(t)}}{p(\mathbf{x}_k | \Psi^{(t)})} = \frac{\pi_m^{(t)} p(\mathbf{x}_k | \Psi_m^{(t)})}{\sum_{j=1}^M \pi_j^{(t)} p(\mathbf{x}_k | \Psi_j^{(t)})} \quad (10)$$

$$\mathbf{W}_k^{(t)} = \frac{\mathbf{v}_m^{(t)} + D}{\mathbf{v}_m^{(t)} + \delta(\mathbf{x}_k, \mu_m^{(t)}; \Sigma_m^{(t)})} \quad (11)$$

$$\delta(\mathbf{x}_k, \mu_m^{(t)}; \Sigma_m^{(t)}) = (\mathbf{x}_k - \mu_m^{(t)})^T (\Sigma_m^{(t)})^{-1} (\mathbf{x}_k - \mu_m^{(t)}) \quad (12)$$

- M-Step: The mixing proportions π_m are updated by the average of the posterior probabilities of component membership of the mixture. The estimates of μ_m , and Σ_m are also updated.

$$\left\{ \begin{aligned} \pi_m^{(t+1)} &= \frac{1}{N} \sum_{k=1}^N p(m | \mathbf{x}_k, \Psi^{(t)}) \\ \mu_m^{(t+1)} &= \frac{\sum_{k=1}^N p(m | \mathbf{x}_k, \Psi^{(t)}) \mathbf{W}_k^{(t)} \mathbf{x}_k}{\sum_{k=1}^N p(m | \mathbf{x}_k, \Psi^{(t)}) \mathbf{W}_k^{(t)}} \\ \Sigma_m^{(t+1)} &= \frac{\sum_{k=1}^N p(m | \mathbf{x}_k, \Psi^{(t)}) \mathbf{W}_k^{(t)} (\mathbf{x}_k - \mu_m^{(t+1)}) (\mathbf{x}_k - \mu_m^{(t+1)})^T}{N \pi_m^{(t+1)}} \end{aligned} \right. \quad (13)$$

The above two steps are repeatedly implemented for each input until Ψ converges.

B. A modified EM Algorithm

Initialization of EM is a critical issue because EM converges to a local maximum of the likelihood function: the final estimate depends on the initialization. There are several initialization algorithms, such as HCM, LBG (Linde-Buzo-Gray) and fuzzy c-mean (FCM) clustering method.

Fuzzy clustering methods allow objects to belong to several clusters simultaneously, with different degrees of membership. The data set $\mathbf{X}$ is thus partitioned into v fuzzy subsets. In many real situations, fuzzy clustering is more

natural than hard clustering, as objects on the boundaries between several classes are not forced to fully belong to one of the classes, but rather are assigned membership degrees between 0 and 1 indicating their partial memberships. In this case, we used the FCM clustering method for initialization of TMM in each cluster and then the two steps (E-step & M-step) are applied.

The fuzzy K-means clustering algorithm is based on the minimization of the following objective function

$$J(\mathbf{U}, \mathbf{V}) = \sum_{i=1}^N \sum_{j=1}^c u_{ij}^b d^2(\mathbf{x}_i, \mathbf{v}_j) \quad (14)$$

with

$$\begin{aligned} u_{ij} &\in [0, 1], \quad (i = 1, 2, \dots, N; j = 1, 2, \dots, c) \\ \sum_{j=1}^c u_{ij} &= 1, \quad \forall i = 1, 2, \dots, N \\ 0 &< \sum_{i=1}^N u_{ij} < N, \quad (j = 1, 2, \dots, c) \end{aligned} \quad (15)$$

where $\mathbf{U} = (u_{ij})_{N \times c}$ denotes the fuzzy cluster matrix, and u_{ij} is the degree of membership of $\mathbf{x}_j$ in the i^{th} cluster. The fuzzy C-means clustering algorithm is based on the minimization of the following objective function

$$\begin{aligned} J(\mathbf{U}, \mathbf{V}) = & \sum_{i=1}^N \sum_{j=1}^c u_{ij}^b d(\mathbf{x}_i, \mathbf{v}_j) \\ & - \sum_{i=1}^N \lambda_i \left(\sum_{j=1}^c u_{ij} - 1 \right) \end{aligned} \quad (16)$$

The fuzzy set will be determined by using an iterative method where $J(\mathbf{U}, \mathbf{V})$ is successively minimized with respect to fuzzy membership u_{ij} and cluster centroid $\mathbf{v}_j$.

Now we construct the modified EM algorithm for multivariate TMM.

- 1) *Initialization.* Using FCM method, we initialize the parameters of TMM, including initializing the M, π_m, μ_m and Σ_m . Then, at each time step t , we implement the following two steps of EM algorithm, as aforementioned
- 2) E-step;
- 3) M-step.

So far, we can build a TMM statistical model to represent the statistical characteristics of target HRRP.

IV. EXPERIMENTAL RESULT

The main procedure of radar HRRP target recognition based on the modified algorithm for the proposed statistical model will be described in the following.

A. Data Description

This results presented in this paper are based on simulated airplane data [6]. The time-domain data of three groups of targets, namely, a Large-sized, a Middle-sized and a Small-sized target groups, consisting of 8 complex targets

(aircrafts: $T_1, T_2, T_3, T_4, T_5, T_6$, and cruise missiles T_7, T_8) are simulated using the Radar Target Backscattering Simulation software [6]. The simulation parameters are as follows: The central radar frequency is 10.0 GHz with a chirped pulse waveform with a bandwidth of 1.0GHz. The signatures for each target are sampled over the azimuthal range of 0 to 180° from nose-on, with an azimuthal interval of 0.05°. The antenna polarization is H-H.

In order to compare the identification rates in different aspect area, we select four sets, namely, 6°, 30°, 60° and 90° as the recognition sets. Our target recognition is of 8 targets in each aspect set. We divide the simulated data into two subsets: the odd series are used for training, and the even series are used for testing. We use training data from different target-aspect angles to establish different frame models.

Some preprocessing needs to be applied to the range profiles (both training and test) to ensure that the targets are consistently aligned. In this paper, this is done by time-shift compensated using the slide correlation processing.

To investigate the noise performance of the proposed identification method, Gaussian noise are generated and added to the simulated target signature data with certain signal-to-noise ratio (SNR) defined by

$$SNR = 10 \times \log_{10} \left(\frac{E_s}{\sigma_n^2} \right) \quad (17)$$

where E_s is the total energy of the frequency-domain data, σ_n^2 is the variance of the additive noise.

B. Recognition Experiments

Here is the identification result of 8 radar targets. Table I and Table II show the correct recognition rates of TMM with the modified EM algorithm in different aspect sets, and with $SNR = 15$ dB and $SNR = 20$ dB, respectively. Obviously, the correct recognition rates P_d increase as the aspect set gets smaller when the SNR is fixed. It can also be seen that, for fixed aspect set, the correct recognition rates increase with higher SNR . When $SNR = 15$ dB, in 6° aspect set, the average P_d of 8 targets is 97.88%; whereas, $SNR = 20$ dB, in 90° aspect set, the average P_d of 8 targets is 65.56%.

TABLE I: THE CORRECT RECOGNITION RATES OF TMM WITH $SNR = 15$ DB (%)

Target	Aspect Set (deg)			
	6	30	60	90
T_1	100	86.5	69.25	61.5
T_2	100	97.75	90.5	91.5
T_3	100	100	79	69.25
T_4	100	84.25	74	55.5
T_5	100	82	53.25	40.25
T_6	97.75	82.5	62.5	58.25
T_7	93.25	30	18.25	13
T_8	92	29.5	17.5	15.5
Average recognition rate P_d	97.88	74.06	58.03	50.59

The average recognition rates of TMM, Gaussian mixture model and template matching method under the maximum correlation coefficient criterion (MCC) [1] versus the SNR (6° aspect set) are shown in Fig. 1. From Fig. 1 it is clearly seen that: (1) statistical models have better recognition

performances than the nonstatistical methods; (2) the average recognition rate of the TMM, with $SNR = 15$ dB, is approximately 4 percentage points larger than those of Gaussian mixture model.

TABLE II: THE CORRECT RECOGNITION RATES OF TMM WITH $SNR = 20$ DB (%)

Target	Aspect Set (deg)			
	6	30	60	90
T_1	100	100	89.25	80.5
T_2	100	100	95.5	88.5
T_3	100	100	97	83.5
T_4	100	100	100	90.25
T_5	100	100	75.75	60
T_6	100	100	86.75	81
T_7	100	40	23.25	19
T_8	100	43.75	25.75	21.75
Average recognition rate P_d	100	85.47	74.16	65.56

Fig. 1. The average recognition rates of MCC method, GMM and TMM versus the SNR.

V. CONCLUSION

We have developed a t-mixture model (TMM) approach to radar automatic target recognition, in which echoes of target are modeled by t-mixture distributions. Estimation for the parameters of TMM is achieved by the modified EM algorithm, i.e., fuzzy EM algorithm (FEM). Compared to the Gaussian mixture model, the t-distribution has wider tail than Gaussian distribution and has robustness in the presence of the observations that were polluted by noise. Experimental results for simulated data show that the proposed statistical model not only has better recognition performance but also is more robust to noises than Gaussian mixture model.

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