A Heuristic Method: Differential Evolution for Harmonic Reduction in Multilevel Inverter System

P. Jamuna and C. Christober Asir Rajan

Abstract—In this paper, Differential Evolution (DE) optimization technique is applied to determine optimum switching angles for cascaded multilevel inverter topology with non equal dc sources for eliminating pre specified order of harmonics while maintaining the required fundamental voltage. This paper discusses briefly an efficient Differential Evolution algorithm (DE) that reduces significantly the computational burden resulting in fast convergence. An objective function describing a measure of effectiveness of eliminating selected order of harmonics while controlling the fundamental component is derived. This technique can be applied for any number of levels; as an example in this paper 7-level inverter with different modulation indices and switching angles are reported. Then, these angles are used in simulation to validate the results.

Index Terms— RF MEMS, shunt switch, dc and rf characteristics, pull-in voltage, insertion loss, isolation, spring constant, young's modulus, poisson's ratio.

I. INTRODUCTION

A multilevel inverter is a power electronic system that synthesis a desired voltage output from several levels of dc voltage as input. The cascaded multilevel inverter consists of a series of H-bridge inverter units [1]-[3]. Multilevel inverters also have several advantages with respect to hard-switched two-level pulse width-modulation (PWM)[4], adjustable-speed drives (ASDs). Motor damage and failure have been reported by industry as a result of some ASD inverters' high-voltage change rates (dv/dt), which produced a common-mode voltage across the motor windings. High-frequency switching can exacerbate the problem because of the numerous times this common-mode voltage is impressed upon the motor each cycle. The main problems reported have been motor bearing failure and motor winding insulation breakdown because of circulating currents, dielectric stresses, Voltage surge, and corona discharge. Multilevel inverters generate a staircase waveform. By increasing the number of output levels, the output voltages have more steps and harmonic content on the output voltage and the THD values are reduced. Therefore, they produce high quality output voltage by increasing the level number. However, in some conditions, the harmonic

components on an output voltage wave are required for special applications. Specific harmonic component required for voltage active filter applications. This paper presents the performance of new single phase multilevel inverter where its structure is totally different from some inverter type in literature. The level number can be easily increased [5]-[7]. As a result, voltage stress is reduced and more sinusoidal shaped output voltage waves can be obtained. The proposed inverter works perfectly for reduction of harmonic content and also to maintain specific harmonic content for some applications. In this paper, Differential Evolution algorithm (DE) approach will be presented, which solves the transcendental equations with a simpler formulations and with any number of levels without extensive derivation of analytical expression. A multilevel inverter based on the cascaded converter topology with non-equal dc sources is studied in Fig. 1. An accurate solution is guaranteed even for a number of switching angles that is higher than other techniques would be able to calculate for given equations.

Fig 1. Single phase structure of a cascaded multilevel inverter

II. ASYMMETRICAL MULTILEVEL INVERTER

Various multilevel inverters structures are reported in the technical literature, such as diode-clamp multilevel inverters(neutral-clamp), capacitor-clamp multilevel inverter(flying capacitor), cascaded multi-cell with separate dc sources and hybrid inverters that are derived from the above mentioned topologies with the aim to reduce the amount of semiconductor elements[3]. In this paper, the cascaded 7-level inverter configuration is implemented. It is formed by connecting several single phase H-bridge converters in series as shown in Fig. 1. An asymmetrical

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multilevel inverter can be defined as a multilevel converter fed by a set of dc-voltage source capacitor where at least one of them is different to the other one. The seven level inverter with different dc source, the circuit is shown in Fig. 2.

Fig. 2. Seven level inverter circuit

Each converter generates a square wave voltage waveform with different duty ratios, which together form the output voltage waveform as in Fig 3. A three phase configuration can be obtained by connecting three of these converters in Y or Δ . For harmonic optimization, the switching angles α_1 , α_2 and α_3 shown in Fig. 3 have to be selected so that certain order harmonics are eliminated [4], [6], [8]. The multilevel inverter used is Asymmetrical with binary configuration where levels are identified using,

$$\text{Level}, l = 2^{M+1} - 1$$

where, M = number of dc sources or number of H-bridge connected.

Fig. 3. Generalized stepped output waveform

III. PROBLEM FORMULATION

The output stepped voltage waveform is analyzed using Fourier theory is shown below.

$$V_{out}(\theta) = a_0 + \left[\sum_{n=1,2,\dots}^{\infty} a_n \cos(n\theta) + b_n \sin(n\theta) \right] \quad (1)$$

Considering the output waveform characteristics of odd and half-wave symmetry, now the equation (1) becomes,

$$V_{out}(\theta) = \sum_{n=1,3,5}^{\infty} b_n \sin(n\theta) \quad (2)$$

where b_n is given by

$$b_n = \sum_{n=1,3,5}^{2N-1} \frac{4V_{dc}}{n\pi} (V_1 \cos(n\alpha_1) + V_2 \cos(n\alpha_2) + \dots + V_M \cos(n\alpha_N)) \quad (3)$$

where,

$n = 1, 3, 5, \dots, 2N-1$ (odd harmonics only)

N = number of switching angles per quarter cycle, and

M = number of dc sources.

Equation (3) has N switching angles where solutions for such equations can be obtained by equating $N-1$ harmonics to zero and assigning specific value to fundamental component. Solutions can be obtained through many iterative techniques such as Newton raphson method, Walsh function, and resultant theory or through minimization approach. In order to precede with minimization technique objective function this reduces the pre- specified harmonics to zero while maintaining the fundamental component is defined. Objective function is defined as,

$$F(\alpha_1, \alpha_2, \dots, \alpha_N) = \left[\sum_{k=1}^N V_1 \cos(\alpha_k) - A_0 \right]^2 + \left[\sum_{k=1}^N V_2 \cos(3\alpha_k) \right]^2 + \dots + \left[\sum_{k=1}^N V_M \cos((2N-1)\alpha_k) \right]^2$$

where as for 7-level inverter only 3rd and 5th harmonics are considered. Since switching angles are $N=3$, $N-1$ harmonics (3, 5) are equated to zero. Thus the objective function becomes,

$$F(\alpha_1, \alpha_2, \alpha_3) = \left[\sum_{k=1}^N V_1 \cos(\alpha_k) - A_0 \right]^2 + \left[\sum_{k=1}^N V_2 \cos(3\alpha_k) \right]^2 + \left[\sum_{k=1}^N V_3 \cos(5\alpha_k) \right]^2 \quad (4)$$

where α_k is the k^{th} switching angle,

$$A_0 = \frac{M \times m_i \times \pi}{4}$$

$$m_i = \frac{H_1}{M \times V_{dc}}$$

m_i is the modulation index which lies between $(0 < m_i < 1)$, and H_1 is the fundamental component.

The optimal switching angles are obtained by minimizing the equation (4) by equating to further constraints of equation (5) and this helps to eliminate the certain order of harmonics.

$$(0 \leq \alpha_1 \leq \alpha_2 \leq \dots \leq \alpha_N \leq \frac{\pi}{2}) \quad (5)$$

IV. DIFFERENTIAL EVOLUTION (DE)

Differential Evolution (DE) algorithm is a new heuristic approach mainly having three advantages; finding the true global minimum regardless of the initial parameter values, fast convergence, and using few control parameters. DE algorithm is a population based algorithm like genetic algorithms using similar operators; crossover, mutation and selection [9]. In this work, we have compared the performance of DE algorithm to that of some other well known versions of genetic algorithms. It was observed that the convergence speed of DE is significantly better than genetic algorithms [10]. Therefore DE algorithm seems to be a promising approach for engineering optimization problems.

Evolutionary algorithm is explained through block diagram shown below (see Fig. 4):

Fig. 4. General procedure for evolutionary algorithm

Mutation is the process which is performed prior with the continuation of recombination which is also said to be crossover. Finally best chromosomes are selected through selection or reproduction process [11].

The steps involved in differential evolution technique are explained below:

- 1) Initialize the maximum number of iterations $n_{\max}$, minimum cost value ε .
- 2) Generate initial population randomly within the range.
- 3) Set iteration $n=0$.
- 4) Calculate cost function $f(x^n)$ and store the angles as base vector.
- 5) Form two duplicate matrices of switching angles by shuffling the rows (p_{m1}, p_{m2}).
- 6) Mutate those values and form a trial vector V^{n+1} by the given formula,

$$V^{n+1} = \text{base value} + F \cdot (p_{m1} - p_{m2})$$

where, F = scalar factor which should be less than 1.

- 7) Perform crossover, assume $0 < p_c < 1$ where baby Y^{n+1} is formed as per,

$$Y^{n+1} = \begin{cases} V^{n+1}, & \text{if } \text{rand} < p_c, \text{ base values, otherwise.} \end{cases}$$

With the formed values calculate cost function and compare with previous function values.

$F(x^{n+1}) < F(x^n)$, if so replace the original values with newly generated values.

8) From step 4 to step 8 will be repeated until termination criteria is met, $F(x^n) < \varepsilon$ or $n > n_{\max}$.

Flowchart (see Fig. 5):

Fig. 5. Flowchart for differential evolution

Fig. 6. Output voltage waveform of seven level single phase inverter

V. SIMULATION AND RESULTS

To implement the proposed DE, a program was developed using the software package MATLAB 7.5 [12]. With the newly proposed inverter system, the algorithm is applied where some specified order of harmonics is reduced. It should be noted that the level of the dc sources are non-equal and can be measured. Furthermore, simulations presented concern a single phase system. However, this does not reduce the way the algorithm can be applied in the three-phase system. The location of harmonics to be eliminated vary between the single and three-phase case since the triplen harmonics can be eliminated by the converter structure and

there is no need to be included in the elimination process.

The algorithm was used to find the switching angles for certain modulation index range within which solution exists, i.e., $0.75 \leq m_i \leq 0.95$. Fig. 6 shows the variation of switching angle vs. the modulation index. The conventional optimization technique of Newton-Raphson (NR) has been described in much literature. In differential evolution algorithm we have set 30 iterations and the degree of accuracy or halting conditions is considered to be 0.00001. This differs from the conventional method for the computational times and as well as in THD values. This exhibits clearly that the proposed technique is more than two times faster than the conventional method Newton-Raphson.

In this paper a quality factor chosen as a performance index to indicate the usefulness and effectiveness of the method implemented. The total harmonic distortion (THD) is a useful factor considered for the evaluation of the inverter performance and therefore THD is considered in this paper. After running the matlab coding for DE of modulation index 0.8, the output switching angle is given below:

$$\begin{aligned}\alpha_1 &= 7.3593^\circ \\ \alpha_2 &= 29.4584^\circ \\ \alpha_3 &= 54.4846^\circ\end{aligned}$$

Fig. 7. Modulation index vs. switching angle

Fig. 8. Spectrum of output voltage waveform

Fig 9. Modulation index vs. total harmonic distortion

Here 3rd and 5th harmonics are 0.03% and 0.06% with 100% fundamental voltage. The output waveform for 7-level inverter is shown in the Fig. 6. Fig. 7 shows the variation of switching angle vs. the modulation index

Fig. 8. shows the spectrum of output waveform where it has been observed that low frequency harmonics (3rd and 5th) has been reduced. Fig. 9 depicts the variation of THD with modulation index. Table I shows the switching angles and THD values for various modulation indices.

TABLE I: SIMULATED OUTPUTS FOR VARIOUS MODULATION INDICES

Modulation index(mi)	α_1	α_2	α_3	THD (%)	3 rd order harmonics (%)	5 th order harmonics (%)
0.75	5.2319	30.4231	53.9134	10.26%	0.48%	0.44%
0.8	7.3593	29.4584	54.4846	9.24%	0.03%	0.06%
0.85	14.3272	25.4782	57.6222	11.13%	0.33%	0.61%
0.9	10.7037	27.5339	55.7440	8.99%	0.26%	0.02%
0.95	8.6268	28.8818	55.0062	9.20%	0.41%	0.17%

VI. CONCLUSION

To generate optimal switching angles in order to eliminate a certain order of harmonics, a new technique is introduced in this paper. The differential evolution(DE) is proposed to overcome the computational burden and to ensure the accuracy of the calculated angles. The algorithm was developed using MATLAB software and is run for a number of times independently to ensure the feasibility and the quality of the solution. Only one set of solutions is documented and plotted in this paper. The comparison of the results in this paper to similar work in the literature shows that the DE approach for the harmonic optimization of multilevel inverters work properly. The simulated output of switching angles for various modulation index is shown. DE can be applied to any problem where optimization is required; therefore, it can be used in many applications in power electronics.

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