The Analysis of the Convergence of Newton-Raphson Method Based on the Current Injection in Distribution Network Case

Tan Tingting, Li Yang, Li Ying, and Jiang Tong

***Abstract*—This paper applies the Newton-Raphson method based on current injection into the case of distribution network. Firstly, the correction equations of these two methods have been derived and compared. The Jacobian matrix of the traditional Newton-Raphson must be recalculated in each iteration, while the Newton-Raphson method based on current injection only need recalculate the diagonal elements of its Jacobian matrix which mainly consists of admittance matrix's elements. It reduces the computation and makes the programming easier as well. Based on these, their convergence properties have been derived. Both of them have the quadratic convergence, the only difference is the coefficients. IEEE11 system has been used to test this method, and compared with the traditional Newton-Raphson method. The results show the Newton-Raphson (N-R)method based on current injection reliable and effectively.**

***Index Terms*—Current injection, newton-raphson, distribution network.**

I. Introduction

The essence of the load flow calculation is to solve a set of nonlinear equations. Gauss and Newton method are two common methods of solving nonlinear equations, the power flow algorithm based on these two methods have a great influence on the development of the calculation in power system [1].

Newton method and fast decoupled load flow (FDLF) are the most preferred method in power flow calculation. But in the distribution network, because of the high ratio of R/X, it is hard for the FDLF to converge [2]. When the distribution network is overloading, the voltages drops seriously, which may influence the convergence of Newton-Raphson method [3]. What's more, the high ratio of R/X may lead to the result that the principal diagonal are not dominant, which may make the algorithm not converge [4]. Taking the characteristics of the distribution network into account, scholars have proposed some improved algorithms based on the traditional methods. But most of them increase the computational complexity, and the effects are not ideal [5]. The forward/backward sweep method is the most common algorithm for the distribution network. When the network is radial, it can achieve the best convergence result However, if the network is meshed or has PV nodes, the algorithm meets difficulties [6-8].Newton-Raphson method based on current injection can improve the convergence of the traditional N-R method. It can formulate the Jacobian matrix easily for both Cartesian and polar coordinates and reduce the computation [9-10]. It can also extend to the three-phase load flow calculation easily [11]. This paper will analyze the convergence of the N-R method based on current injection and traditional N-R method, and takes use of the distribution network to test these two algorithms. The results will be compared to show the advantages of the N-R method based on current injection.

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II. The Correction Equation of the Current Injection Method

The mathematical model of the Current injection method is built on the basis of the traditional N-R method. To make a simple change of the original nonlinear equation $f(x)=0$, its deformation is $\frac{f(x)}{x}=0$ ($x \neq 0$). The correction equation for the traditional N-R method is: $f(x)=f(x_0)+f'(x_0)\Delta x$, while the current injection method's equation is $\frac{f(x)}{x}+(\frac{f'(x)}{x}-\frac{f(x)}{x^2})\Delta x=0$.

Applying this model into load flow calculation, that is to solve the equation $\Delta(\frac{S}{U})=0$ instead of solving $\Delta S=0$. Divided by U for both sides of the nodal power equation $S=\overset{*}{U}\overset{*}{Y}U$, that is:

$$\overset{*}{U}\overset{*}{Y}=\overset{*}{I}=\left[\frac{S}{U}\right] \tag{1}$$

Making Taylor expansion for both sides of the equation, ignoring quadratic and higher order terms, it can be as follows:

$$\frac{S}{U}-\frac{S}{U^2}\Delta U=\overset{*}{U}\overset{*}{Y}+\overset{*}{Y}\overset{*}{\Delta U} \tag{2}$$

Transpose the equation, that is:

$$\frac{S}{U}-\overset{*}{U}\overset{*}{Y}=\frac{S}{U^2}\Delta U+\overset{*}{Y}\overset{*}{\Delta U} \tag{3}$$

Because $\overset{*}{Y}\overset{*}{\Delta U}=Y'\Delta U$ the equation can be

$$\Delta(\frac{S}{U})=(\frac{S}{U^2}+Y')\Delta U \tag{4}$$

It is the correction equation of the current injection

method and $(\frac{S}{U^2}+Y')$ is the Jacobian matrix, solving the equation (4) can get the results.

The Jacobian matrix of the traditional N-R must be recalculated at each iteration, which leads to lager computation and increases the time of computation. From (4), it is can be seen that the upper triangular and lower triangular elements are mutual attendance, which Remain unchanged at each iteration. The diagonal elements are the sum of the self-admittance and Diagonal matrix. Therefore, only the diagonal need to be calculated at each iteration, which can reduce the computation.

III. Analysis of the Convergence of Newton-Raphson and Current Injection Method

For any equation, it can do the Taylor expansion at one value, having the following formula:

$$f(x)=f(x^{(n)})+f'(x^{(n)})\Delta x^{(n)}+\frac{f''(\xi)}{2}(\Delta x^{(n)})^2 \tag{5}$$

The deviation can be expressed as follows:

$$\Delta x^{(n)}=\frac{f(x)-f(x^{(n)})}{f'(x^{(n)})}-\frac{f''(x^{(n)})}{2f'(x^{(n)})}(\Delta x^{(n)})^2 \tag{6}$$

The N-R algorithm ignoring quadratic and higher order terms, the deviation can be expressed as follows:

$$\Delta' x^{(n)}=\frac{f(x)-f(x^{(n)})}{f'(x^{(n)})} \tag{7}$$

At the *N* times iteration

$$\begin{aligned} x^{(n+1)} &= x^{(n)}+\Delta' x^{(n)} \\ &= x^{(n)}+\frac{f(x)-f(x^{(n)})}{f'(x^{(n)})} \\ &= x^{(n)}+\Delta x^{(n)}+\frac{f''(x^{(n)})}{2f'(x^{(n)})}(\Delta x^{(n)})^2 \\ &= x+\frac{f''(x^{(n)})}{2f'(x^{(n)})}(\Delta x^{(n)})^2 \end{aligned}$$

Transpose the equation, it can be:

$$x^{(n+1)}-x=\frac{f''(x^{(n)})}{2f'(x^{(n)})}(\Delta x^{(n)})^2 \tag{8}$$

That is:

$$\Delta x^{(n+1)}=\frac{f''(x^{(n)})}{2f'(x^{(n)})}(\Delta x^{(n)})^2 \tag{9}$$

Set up $g(x)=\frac{f(x)}{x}$, the convergence equation of current injection is as follows:

$$\Delta x^{(n+1)}=\frac{g''(x^{(n)})}{2g'(x^{(n)})}(\Delta x^{(n)})^2 \tag{10}$$

Current injection method has the same quadratic convergence rate just as the traditional N-R method. As the equations of them are different, the convergence rate may be different. However, as the main factor of the convergence is the Quadratic Term, not its coefficient, these two methods have no significant difference.

IV. Analysis of Cases

This paper takes IEEE11 system as an example, chooses the Visual Studio 2010 development platform, takes use of C++ to programming program, and the accuracy of the case ε is 10^{-5}.

A. The Comparison of the Results of the Current Injection and N-R

For IEEE11 system , taking the same initial value of 1.0 and with the same accuracy , both of them convergent after 9 times iterations, but have different results ,which shows in the current injection -type Newton method than Newton-Raphson convergence of the number , are nine times , but converge to different results, and table below shows the results.

TABLE I: The Results of These Two Methods

N-R based on current injection method	N-R
1.024	1.024
1.05844	1.05616
1.04803	1.04536
1.03391	1.02992
1.03702	1.03390
1.05400	1.04993
0.81516	0.77814
0.91653	0.86374
1.20367	1.13319
0.82394	0.75440
1.08174	0.985624

B. The Analysis of the PV Curve

Adjusting the active power of 11 nodes from 0 to its limit, the Newton - Raphson method and the current injection method are used to calculate the solutions respectively with the same initial value 1.0, the results are in the table below:

TABLE II: The Results of the PV Curve by Newton-Raphson and Current Injection Method

Active Power	The voltage amplitude 1	The voltage amplitude 2
0	0.195739	1.84611
0.025	0.221861	1.78549
0.05	0.281574	1.71746
0.075	0.366693	1.63934
0.1	0.474712	1.54615
0.125	0.613097	1.42657
0.15	0.825231	1.23526
0.158*	0.985624	1.08282

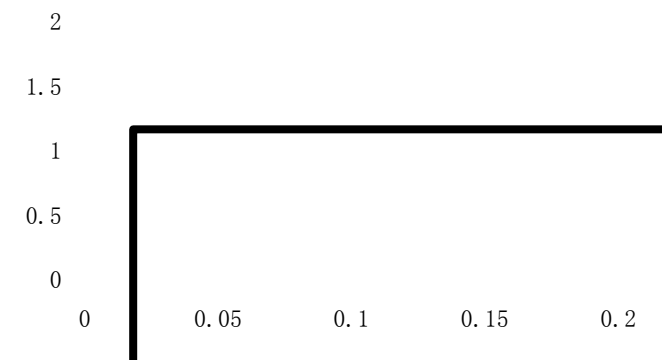

Fig. 1. PV curve by by Newton-Raphson and Current Injection method

The Table II shows that the solutions of the current injection are greater than those of the traditional N-R when start the iteration at the same initial conditions. The data are

used to draw the PV curve. The results of the traditional N-R method is related to the lower portion of the PV curve which are unstable solutions in theory ,while the solutions of the current injection are corresponding to the upper portion of the curve which are stable. In addition, the picture shows that the results of the IEEE11 system are close to the inflection point of the curve (the marked dashed line in picture). Therefore the IEEE11 system is ill-conditioned.

V. Conclusion

Newton-Raphson method based on currrent injection has the same quadratic convergence rate the same as the traditional N-R method. It can convergent fast. However, the current injection method has simple Jacobian matrix and smaller computation in each iteration, which can make the programming easier and reduce the time of the computation. The distribution network IEEE11 case shows that with the same initial values, the current injection methods can convergent to the upper part of the PV curve, that is the stable solution of the system. The IEEE11 case indicates the current injection method is more reliable than the traditional Newton-Raphson.

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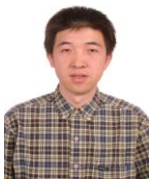

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