Optimization of Distribution Network Distributed Reactive Compensation

Cheng Xiaoxiao, Sun Qian, Yin Junhe, and Guo Jianli

***Abstract*—In order to solve the optimization of distribution reactive compensation point and capacity, a double optimized model is proposed, which is suitable for reactive compensation optimization with random load distribution or random network structure. For the compensation position and capacity decision variables, the optimized model is described as two layers of optimization with constraint. The outer one is the capacity optimization at determined location, and the inlayer is the location optimization. By introducing Lagrange multiplier, the mixed integer nonlinear optimization is deduced to a linear one that can be easily solved by Gaussian elimination method. For illustration, an application of ten 10kV rural feeders is utilized to show the feasibility of the double optimized model in solving the optimization of distribution reactive compensation point and capacity. Empirical results show that the model can give the optimized result for different number of capacitor installation, and the result with highest line loss decrement will be used as the final decision. This research provides scientific theoretical basis for reactive compensation and plays a vital role in reactive compensation equipment installation and line loss management.**

***Index Terms*—Distribution line, distributed reactive compensation, optimized configuration, decision-making of loss reduction.**

I. Introduction

The reactive power compensation for distribution network, as the supplement of substation compensation can effectively improve the power factor, reduce line loss, improve the end voltage, ensure the quality of power supply, also bring good economic benefits for enterprise, has received extensive attention. The distributed reactive compensation, installing power capacitors on feeders, is the main distribution network compensation mode at home and abroad [1], but different installed location and different installed capacity, the benefit is different. With the application of reactive power compensation distribution increase gradually, how to choose appropriate reactive compensation location and compensation capacity to make the maximum benefit with less cost become people's research target. And the optimization of distributed reactive compensation of distribution network was raised [2], [7], [8].

At present, the decision of the best compensation capacity and the best position in actual distribution reactive compensation, usually in accordance with ideal situations, such as, the reactive load along the road distributed uniformly, increasing, diminishing distribution or as isosceles distribution, and so on [2], [9]. This method has clear results, simple calculation, and has a certain engineering practical value. But the actual reactive load distribution is more complex, which is different from the ideal situation. So, in accordance with ideal situations to premise reactive compensation configuration optimization formula may be not satisfied. To study a more general distributed reactive compensation configuration optimized method is needed.

This paper studies several kinds of typical optimal allocation of reactive compensation configuration with ideal load distribution. Then it details the distributed reactive compensation optimized mathematical model, which is applied to any load distribution or distribution network structure, and gives the effective algorithm. At last, the paper introduces the practical application of the research of the model and the algorithm.

II. The Reactive Compensation Optimize Configuration with Ideal Load Distribution

TABLE I: The results of reactive compensation in ideal load distributed situation

Distribution	Capacitor		location	Capacity
Evenly	one		From first end 2/3	2/3 of total reactive load
	two	#1	From first end 2/3	2/5 of total reactive load
		#2	From first end 4/5	2/5 of total reactive load
Increasing	one		From first end 77.5%	80% of total reactive load
	two	#1	From first end 54%	19.3% of total reactive load
		#2	From first end 86%	25.9% of total reactive load
Decreasing	one		From first end 44.2%	62.6% of total reactive load
Isosceles	one		From first end 55.4%	79.6% of total reactive load

The ideal load distribution is refers to the reactive power load distributed along the line meet a kind of ideal regular distribution, for example, in any point the road reactive load

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is equal, named uniform distribution, the reactive load from the first end increasing or decreasing, named increasing or decreasing distribution, and so on. This is an abstract of the actual load distribution, and in such a hypothesis premise the analytical expressions of the optimal location and capacity can be deduced, which can get the best reduce loss effect. And the results are showed in Table I and Fig 1, which can be chose in practical projects [3], [4], [6].

When the actual power distribution is different from the ideal situation, using the results to guide the reactive compensation configuration, the effect may be not beautiful. It needs to study a more general reactive compensation configuration optimized method.

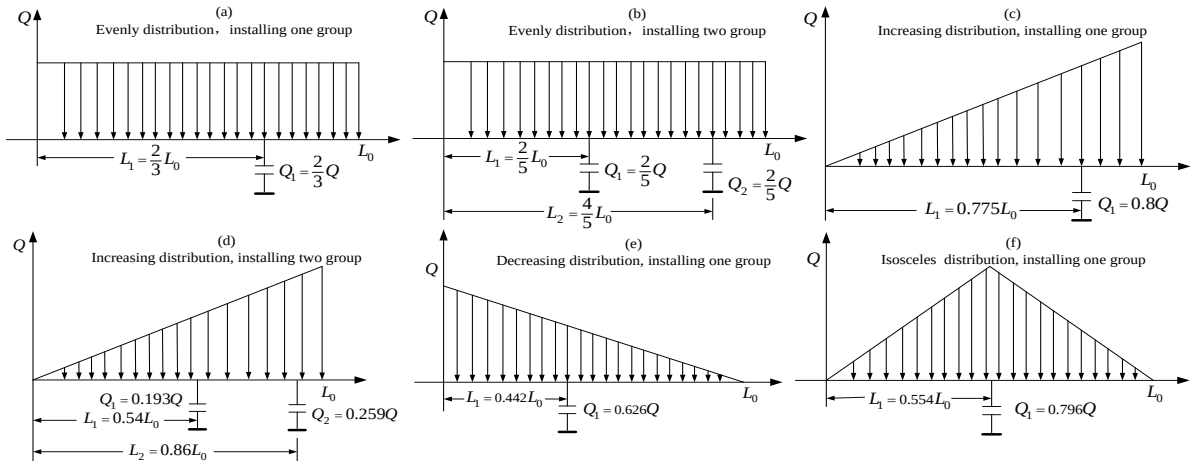

Fig. 1. The results of reactive compensation in ideal load distributed situation

III. The Reactive Power Compensation Optimization in Actual Load Distribution

A. The Mathematic Model

The optimization of distribution network distributed reactive compensation is distributed as a mixed integer nonlinear optimization problems, which is to determine the reactive compensation position and capacity with some constraints [5]. Therefore, the compensation position and capacity are the two decision variables. Its mathematical model is a two layers optimized problem with constraint. First is the capacity optimization at determined location, second is the distribution optimization.

$$obj.\quad \min_{\mathbf{x}_C}\left[\min_{\mathbf{Q}_C} F\left(\mathbf{Q}_C,\mathbf{x}_C\right)\right]$$

$$s.t.\quad \begin{cases} \sum_{j=1}^{n} Q_{Cj} = Q_{C\Sigma} \\ Q_{Cj\min} \le Q_{Cj} \le Q_{Cj\max} & \left(j=1,2,\cdots,n\right) \\ x_j \in N & \left(j=1,2,\cdots,n\right) \end{cases} \tag{1}$$

where $\mathbf{Q}_C=[Q_{C1},Q_{C2},\cdots,Q_{Cn}]$ is a row vector of compensation capacity; $\mathbf{x}_C=[x_1,x_2,\cdots,x_n]$ is a row vector of location; $F\left(\mathbf{Q}_C,\mathbf{x}_C\right)$ is the optimal objective function, that can be the line loss, years operation cost, expenses, etc. $Q_{C\Sigma}$ is the total compensation capacity, which is determined according to the objective power factor and the parameters of distribution capacity; $Q_{Cj\min}$ and $Q_{Cj\max}$ is the upper and lower limit of reactive compensation capacity at point x_j; N is reactive compensation position optional set.

B. The Capacity Optimization

The capacity optimization at determined location is described as

$$obj.\quad \min_{\mathbf{Q}_C} F\left(\mathbf{Q}_C,\mathbf{x}_C^s\right)$$

$$s.t.\quad \begin{cases} \sum_{j=1}^{n} Q_{Cj} = Q_{C\Sigma} \\ Q_{Cj\min} \le Q_{Cj} \le Q_{Cj\max} & \left(j=1,2,\cdots,n\right) \end{cases} \tag{2}$$

And, $\mathbf{x}_C^s=[x_1^s,x_2^s,\cdots,x_n^s]$ is the determined location. The objective function can be line loss, years operation cost, expenses, etc. Set the line loss here, other forms of the objective function is similar to the process.

The parameters of the line section i are shown in Fig. 2.

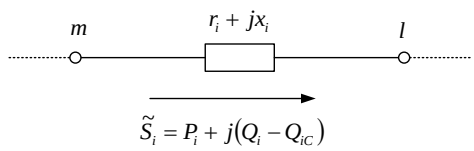

Fig. 2. The line section i in distribution network

P_i is the sum load of downstream nodes of section i; Q_i is the sum reactive load of downstream nodes of section i (not including the reactive compensation). Q_{iC} is the sum capacitor capacity of downstream nodes

$$Q_{iC}=\sum_{j=1}^{n}\alpha_{ij}Q_{Cj} \tag{3}$$

α_{ij} is a 0-1 variable, it takes 1 when the downstream points contain capacitor, otherwise takes 0.

The line loss of section *i* is distributed as ΔP_i [10]

$$\Delta P_i = \frac{P_i^2 + \left(Q_i - Q_{iC}\right)^2}{V_N^2} \cdot r_i \tag{4}$$

The total section is sated as L, so, The line loss is ΔP_Σ, function (2) is specific described as

$$\begin{aligned} &obj. \quad \min_{\mathbf{Q}_C} \Delta P_\Sigma = \sum_{i=1}^{L} \frac{P_i^2 + \left(Q_i - Q_{iC}\right)^2}{V_N^2} \cdot r_i \\ &s.t. \quad \begin{cases} \sum_{j=1}^{n} Q_{Cj} = Q_{C\Sigma} \\ Q_{Cj\min} \le Q_{Cj} \le Q_{Cj\max} \quad (j = 1, 2, \cdots, n) \end{cases} \end{aligned} \tag{5}$$

The inequality constraint is considerate in transfinite. Introducing Lagrange multiplier, equivalent equation of (5) can be structured as follows

$$obj. \quad \min_{\mathbf{Q}_C, \lambda} F = \Delta P_\Sigma + \lambda \left(\sum_{j=1}^{n} Q_{Cj} - Q_{C\Sigma} \right) \tag{6}$$

According to the necessary condition of extreme an equation group is obtained

$$\begin{cases} \sum_{j=1}^{n} a_{kj} Q_{Cj} + \lambda = b_k \quad (k = 1, 2, \cdots, n) \\ \sum_{j=1}^{n} Q_{Cj} = Q_{C\Sigma} \end{cases} \tag{7}$$

where,

$$\begin{cases} a_{kj} = \sum_{i=1}^{L} \frac{2 r_i \alpha_{ij} \alpha_{ik}}{V_N^2} \\ b_k = \sum_{i=1}^{L} \frac{2 Q_i r_i \alpha_{ik}}{V_N^2} \end{cases} \tag{8}$$

Obviously, equation group (8) is a linear one that can be easily solved by using the Gaussian elimination method.

There is a lot of calculation in the optimization. Therefore, the calculation efficiency is very important. From above, it can be seen, the optimization problem (5) is deduced to (6), then according to the extreme conditions, to a problem of solving linear equation group. It is very simple and efficient.

C. The Distribution Optimization

The purpose of distribution optimization is to determine the optimal compensation location.

It is supposed that there are m points in optional location set N. If n points are chosen to install capacitors, therefore, the reactive compensation installation scheme contains combinations. For each combination plan, doing the capacity optimization in section B, the corresponding results were got. Finally sorting all the results, the optimal one which could make the lowest line loss is obtained.

As mentioned above, the capacity optimization was deduced to a problem of solving linear equation group, and the method is simple and effective. So, the enumeration method is adopted in the distribution optimization.

For the compensation optimization has been converted to two layers optimization and the algorithm was simple and effective, so the model and optimization is applied to any load distribution and arbitrary distribution network structure.

IV. Application Examples

Based on the optimization mathematical model and algorithm, the corresponding graphical calculation software has been developed. With the optimization results, some power capacitors are installed on ten 10kV rural feeders which had lower power factor and higher line loss. And the actual operation showed good effect. As shown in Fig 3 and Table II, it is the optimization of a feeder named CHANG 7.the total length is 22.35 km, the conductor type of trunk line is LGJ-120，with a distribution capacity of 4760 kVA. The active power was 1904 kW, and the power factor was 0.83. The objective power factor was set at 0.9, so the reactive compensation total capacity was 358 kvar. The parameters including length and conductor type of each section, nameplate parameters of transformers, and the reactive compensation total capacity were set in the graphical software. Yet, the graph of the feeder had been drawn too. Then the results were marked on the feeder graph automatically, such as Fig. 3.

As shown in Table II, theory line loss rate got an obvious 0.4149 percents decrement, if reactive compensation devices were installed. Also, under the condition of total capacity, two installations made 0.007 percent lower than one, and three points installation made 0.0003 percent lower than two. Then more compensation installations got more decrement of theory line loss rate, but the decreasing rate become inconspicuous, In contrast, equipment maintenance cost increased a lot. Therefore, two installations were selected on CHANG 7 feeder at last.

This work provides scientific and reasonable theory for reactive power optimization of distribution network, and gives a reference for the distribution network loss calculation. Also, it provides the convenience for improving the quality of voltage, energy saving and improving line loss management level.

TABLE II: The Optimization Results of CHANG 7 Feeder

Capacitors	Location	Capacity(kvar)	Theory Line Loss
No			2.358%
One	#65	358	1.943137%
Two	#64	246	1.936948%
	#74	112	
Three	#59	43	1.936119%
	#65	232	
	#74	110	
Four	#59	40	1.935868%
	#64	44	
	#65	173	
	#74	101	

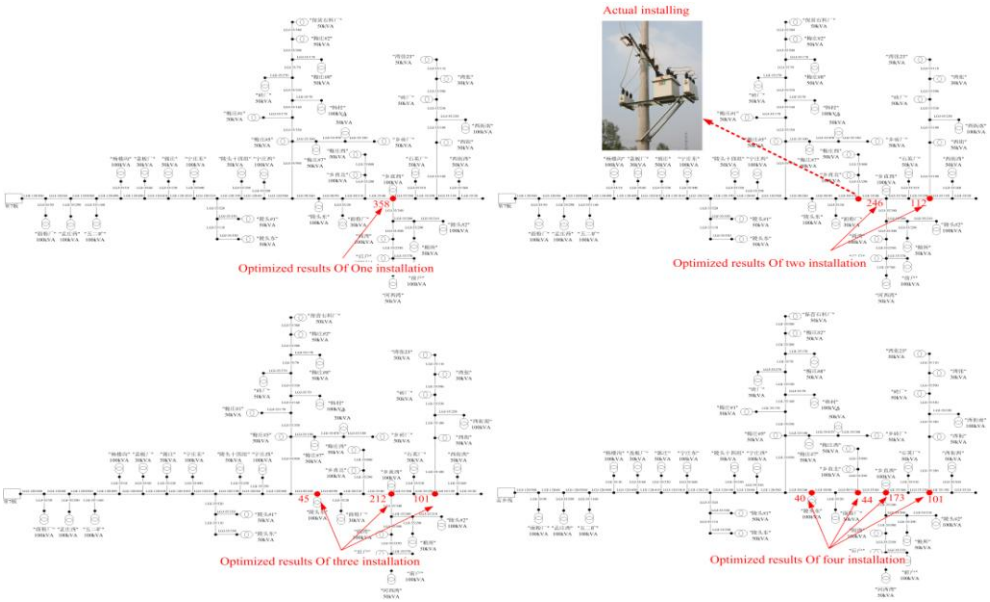

Fig. 3. An instance of distribution network reactive compensation optimized calculation and installation

V. Conclusion

1) For solving distribution network reactive power optimization problem, this paper puts forward the double optimization mathematical model of distribution network distributed reactive compensation, the inner is compensation capacity optimization, the outer layer is the reactive compensation distribution optimization. The model can do distribution reactive compensation optimization with any load distribution and arbitrary distribution network structure forms.

2) By introducing Lagrange multiplier and the necessary condition of extreme, the mixed integer nonlinear optimization problem is deduced to a linear one that can be easily solved by Gaussian elimination method. It is very simple and efficient for computer programming.

3) The model and the algorithm can give different optimized results and loss reduction for different number of capacitor installation. Engineering practice showed that optimized capacitors installation can make line loss rate get an obvious decrement. This research plays an important role in the actual reactive compensation equipment installation of distribution network and line loss management.

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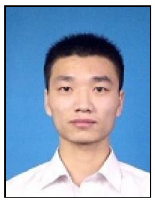

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