

# Performance Analysis of Canadian Power Industry Using Data Envelopment Analysis

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**Abstract**—Canada's electric utility industry has played an important part in the country's history and development. Performance analysis for Canadian power industry is essential. This paper evaluates the performance of 19 Canadian power companies using Data Envelopment Analysis. The results show that the studied Canadian power companies operate very efficiently and company size has a negative effect on the performance.

**Index Terms**—Power industry, data envelopment analysis, optimization, performance analysis.

## I. INTRODUCTION

Canada's electric utility industry has played an important part in the country's history and development. Electricity powers the modern economy, and electricity has been for decades a comparative advantage for Canadian industry. How we address the challenges of the future and the policy decision we make with respect to electricity will inevitably have an impact on the economy [1].

Canada's electric utility industry began as a loosely knit group of investor-owned operations spread across the country. As mass production and massive urbanization shaped 20th century Canada, the economics of electricity supply meant the gradual centralization of portions of the industry under "public" auspices, whether municipally, regionally or provincially [1]. Electric Power Generation, Transmission and Distribution Sector contributed \$28.1 billion to Canada's GDP in 2011 [1]. However, in the twenty-first century, Canadian power industry is facing unprecedented competition and challenges. The California crisis and the Enron debacle make the performance analysis for Canadian power industry essential. This paper presents a performance analysis of Canadian power companies using Data Envelopment Analysis (DEA). DEA is a non-parametric approach to objectively identify best practices in complex operational environments and has been widely used in many real world applications.

The rest of the paper is organized as follows. Section 2 gives a brief review of DEA. Section 3 provides the models and methodology utilized in this paper. Section 4 discusses the sample, gives the DEA result and identifies the factors that affect the performance of Canadian power companies. Finally, the conclusions are presented.

## II. DEA BASICS

Production process can be defined as a process that can turn a set of resources into desirable outcomes by production units. During this process, efficiency is used to measure how well a production unit is performing in utilizing its resources to generate the derived outcomes. Each of the various DEA models seeks to determine which of the  $n$  decision making units (DMUs) define an envelopment surface that represents the best practice, referred to as the empirical production function or the efficient frontier. Units that lie on the surface are deemed efficient in DEA while those units that do not, are termed inefficient. DEA provides a comprehensive analysis of relative efficiencies for multiple input-multiple output situations by evaluating each DMU and measuring its performance relative to an envelopment surface composed of other DMUs. Those DMUs forming the efficiency reference set are known as the peer group for the inefficient units. As the inefficient units are projected onto the envelopment surface, the efficient units closest to the projection and whose linear combination comprises this virtual unit form the peer group for that particular DMU. The targets defined by the efficient projections give an indication of how this DMU can improve to be efficient. Consider  $n$  DMUs to be evaluated,  $DMU_j$  ( $j=1,2,\dots,n$ ) consumes amounts  $X_j=\{x_{ij}\}$  of inputs ( $i=1, 2, \dots, m$ ) and produces amounts  $Y_j=\{y_{rj}\}$  of outputs ( $r=1, \dots, s$ ). The efficiency of a particular  $DMU_0$  can be obtained from the following linear programs (input-oriented BCC model [2]).

$$\begin{aligned}
 \min_{\theta, \lambda, s^+, s^-} \quad & z_0 = \theta - \varepsilon \cdot \bar{1}s^+ - \varepsilon \cdot \bar{1}s^- \\
 \text{s.t.} \quad & Y\lambda - s^+ = Y_0 \\
 & \theta X_0 - X\lambda - s^- = 0 \\
 & \bar{1}\lambda = 1 \\
 & \lambda, s^+, s^- \geq 0
 \end{aligned} \tag{1}$$

Performing a DEA analysis actually requires the solution of  $n$  linear programming problems of the above form, one for each DMU. The optimal variable  $\theta$  is the proportional reduction to be applied to all inputs of  $DMU_0$  to move it onto the frontier. A DMU is termed efficient if and only if the optimal value  $\theta^*$  is equal to 1 and all the slack variables are zero. This model allows variable returns to scale. The dual program of the above formulation is illustrated by:

$$\begin{aligned}
 \max_{\mu, v} \quad & w_0 = \mu^T Y_0 + u_0 \\
 \text{s.t.} \quad & v^T X_0 = 1 \\
 & \mu^T Y - v^T X + u_0 \bar{1} \leq 0 \\
 & -\mu^T \leq -\varepsilon \cdot \bar{1} \\
 & -v^T \leq -\varepsilon \cdot \bar{1} \\
 & u_0 \text{ free}
 \end{aligned} \tag{2}$$

If the convexity constraint ( $\bar{1}\lambda = 1$ ) in (1) and the variable  $u_0$  in (2) are removed, the resulting model is CCR model. The reader is advised to consult the textbook by Cooper, Seiford and Tone [3] for a comprehensive treatment of DEA theory and application methodology.

### III. METHODOLOGY

Some researchers have conducted research on corporation's performance using DEA. Zhu [4] developed a three-input-two-output DEA model to evaluate the

companies' ability to generate the profits and revenues using asset, equity and labour. His analysis is based on the Fortune 500 companies in the U.S. Khoo [5] adopted Zhu's model and developed an operating model based on Du Pont chart to evaluate the Canadian manufacturing companies. This paper utilizes Zhu's approach to evaluate the Canadian power corporation's ability to produce profits and revenues from assets, equity and labour. There are three inputs (total assets, total equity and labour cost) and two outputs (revenue and profit) in the model.

Input orientation was selected for the DEA model in this research. An assumption was made that management is more interested in minimizing the consumption of inputs subject to attaining the desired output level. BCC model is used and scale efficiency issues were examined.

### IV. RESULTS AND DISCUSSIONS

In this analysis, 19 Canadian power companies are included. Table 1 summarizes the statistics of the inputs and outputs for the involved companies.

TABLE I: SUMMARY STATISTICS (CANADIAN DOLLARS IN THOUSANDS)

	Total Asset	Total Equity	Labour Cost	Revenue	Profit
Max	32,136,000	8,393,000	2,756,000	5,471,000	641,000
Min	243,915	83,212	4,268	34,543	4,019
Average	6,545,713	2,012,571	396,664	1,791,609	173,211
Standard Deviation	8,462,251	2,172,424	635,351	1,674,974	189,456

Table I shows that there is significant variation among companies. Therefore, BCC model is a good choice because it can deal with the size effect on the company efficiency. In order to validate the choice of inputs and outputs of the DEA model, correlation coefficients between each pair of input and output variables are calculated. If a very high correlation coefficient is found between any pair of input variables or output variables, this variable may be thought of as a proxy of the other variable and one of them should be excluded from the analysis. If an input variable has very low correlation with all the output variables (or an output variable has very low correlation with all the input variables), it may indicate that this variable does not fit the model. Table 2 shows the correlation coefficients between each pair of input and output variables.

TABLE II: CORRELATION ANALYSIS

	Total Asset	Total Equity	Labour Cost	Revenue	Profit
Total Asset	1.00	0.91	0.90	0.84	0.82
Total Equity	0.91	1.00	0.92	0.88	0.80
Labour Cost	0.90	0.92	1.00	0.76	0.65
Revenue	0.84	0.88	0.76	1.00	0.90
Profit	0.82	0.80	0.65	0.90	1.00

From Table II we can see that there is no evidence of very high correlation between any pair of input or output

variables. In addition, there is no input variable which has very low correlation with any of the output variables (nor between output variable and input variables). This is a reasonable validation of my DEA models.

Table III summaries the DEA results.

TABLE III: DEA RESULTS

	BCC model
<b>Average Score</b>	0.911
<b>Standard Deviation</b>	0.169
<b>Maximum Efficiency Score</b>	1
<b>Minimum Efficiency Score</b>	0.391
<b>Number (and %) of Efficient DMUs</b>	14 (73.7%)
<b>Number of Increasing Returns to Scale (IRS)</b>	7
<b>Number of Constant Returns to Scale (CRS)</b>	10
<b>Number of Decreasing Returns to Scale (DRS)</b>	2

Table III indicates that BCC model identified 91.1% technical efficiency on average. Apparently all the studied Canadian power companies operated very efficiently. However, if all the power companies can perform efficiently, by using the target input values, the involved Canadian power companies could save as much as 8.9% of its resources from a theoretical point of view. In practice, the saving will almost certainly be less. Among all the studied companies, 7 companies operate under IRS, 10 operate under CRS and 2 under DRS. For all the companies

on the efficient frontier, 3 companies operate under IRS, 10 operate under CRS and 1 under DRS. Figure 1 shows the efficiency score distribution.

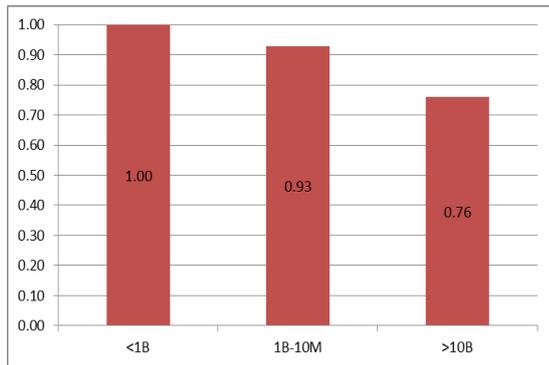


Fig. 1. DEA score distribution

It is obvious from Fig. 1 that the efficiency score distribution is right skewed to higher efficiency score. General speaking, no companies have the efficiency rating below 0.3 and most of the companies have an efficiency rating between 0.9 and 1.0.

The relationship between company’s size and efficiency rating is further investigated. The average technical efficiency scores classified by asset size are illustrated in Fig. 2.

Fig. 2 shows there exists a negative relationship

between studied company’s asset size and efficiency rating.

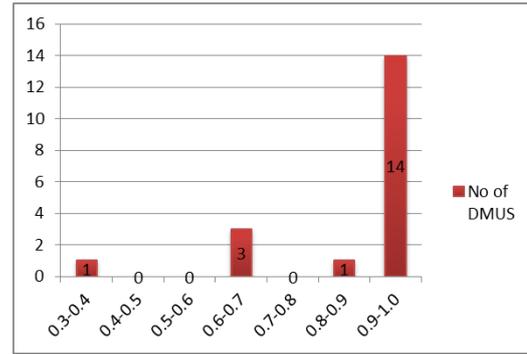


Fig. 2. Relationship between asset size and efficiency rating

On average smaller companies operate more efficiently. This further validates our choice of BCC model which can consider the size effect on efficiency.

One of the most powerful pieces of information that is obtained by a DEA analysis is the set of target values for those DMUs assessed as inefficient. The reference set provides strong indications of what type and amounts of inputs and outputs are needed to make the inefficient units efficient. Table 4 provides the details. For example, in order for DMU<sub>15</sub> to become efficient, it should consume  $X_2 \times 0.282 + X_8 \times 0.718$  ( $X_i$  is the input vector for DMU  $i$ ) amount of input to generate its observed output level.

TABLE IV: REFERENCE SETS

DMU No.	Score	Reference set (lambda)									
		DMU2	DMU3	DMU7	DMU14	DMU15	DMU16	DMU17	DMU18	DMU19	DMU14
1	0.70	0.032	0.070	0.422	0.476						
2	1.00	1									
3	1.00	1									
4	1.00	1									
5	1.00	1									
6	0.69	0.003	0.173	0.588	0.236						
7	1.00	1									
8	1.00	1									
9	1.00	1									
10	0.39	0.163	0.184	0.471	0.182						
11	1.00	1									
12	0.88	0.017	0.035	0.404	0.018	0.527					
13	1.00	1									
14	1.00	1									
15	0.65	0.282	0.718								
16	1.00	1									
17	1.00	1									
18	1.00	1									
19	1.00	1									

V. CONCLUSIONS

This paper adopts DEA approach to evaluate 19 Canadian power companies’ performance. The results show

that Canadian power industry operates very efficiently although some potential savings are identified. In addition, the analysis indicates the asset size has a negative effect on the performance of Canadian power companies.

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