Advanced Intelligent Controller Design for Canonical Nonlinear Systems

R. Ghasemi, *Member, IACSIT*, B. Abdi, *Member, IACSIT*, and S. M. M. Mirtalaei, *Member, IACSIT*

***Abstract*—This paper introduces a new decentralized adaptive fuzzy controller for a class of large-scale nonlinear systems with unknown non-affine subsystems and unknown interconnections represented by nonlinear functions. The fuzzy system is used to represent the controller's structure. The stability of the closed loop system is guaranteed through Lyapunov stability analysis by introducing some adaptive rules derived appropriately. Simulation results easily highlight merits of the proposed controller.**

***Index Terms*—Adaptive Control, Non-affine Nonlinear Systems, Large Scale Systems, Fuzzy Systems, Nonlinear Observer.**

I. Introduction

Fuzzy adaptive controller (FAC) has attracted many researchers to developed appropriate controllers for nonlinear systems because of the following reasons.

As a result of its tunable structure, the performance of the FAC is superior that of the fuzzy controller. Instead of using adaptive controller, FAC can use knowledge of the experts in the controller. In the recent year, FAC has been fully studied as follow:

To use the fuzzy systems for modeling of nonlinear systems and designing the controllers with guaranteed stability are presented in [1]. To model affine nonlinear system and to design stable TS based controllers have been employed in [2]. Designing of the sliding mode fuzzy adaptive controller for a class of multivariable TS fuzzy systems are presents in [3]. In [4], the non-affine nonlinear function are first approximated by the TS fuzzy systems, and then stable TS fuzzy controller and observer are designed for the obtained model. In these papers, modeling and controller has been designed simply, but the systems must be linearizable around some operating points. Designing of the FAC for affine chaotic systems are presented in [5]. To design stable FAC and linear observer for class of affine nonlinear systems are presented in [6], [7]. Fuzzy adaptive sliding mode controller is presented for class of affine nonlinear time delay systems in [8].

[9], [10] are involved stable FAC for class of non-affine nonlinear systems. FAC has been never applied to non-canonical non-affine nonlinear systems. This paper proposes a new method to design an adaptive controller based on fuzzy systems for a class of non-affine nonlinear systems with the following properties: guaranteed stability, robustness against uncertainties and external disturbances and approximation errors, to avoid chattering, and convergence of the output error to zero.

Manuscript received October 9, 2012; revised November 25, 2012.

The authors are with Department of Electrical Engineering, Damavand Branch, Islamic Azad University, Damavand, Tehran, Iran (e-mail: rezaghasemi@damavandiau.ac.ir)

II. Preliminaries

Considering the following large scale non-affine nonlinear system:

$$\begin{cases} \dot{x}_l = f_l(\underline{x}) \quad l = 1, 2, ..., n-1 \\ \dot{x}_n = f_n(\underline{x}) + g(\underline{x})u + d(t) \\ y = C^T \underline{x} \end{cases} \tag{1}$$

where x_l declares l^{th} state, n is number of the state, $\underline{x} = [x_1, \ldots, x_n]^T \in R^n$ is the state vector of the system which is assumed available for measurement, $u \in R$ is the control input, $y \in R$ is the system output, $f_l(\underline{x})$'s, $g(\underline{x})$ are unknown smooth nonlinear function, C are known Constant matrix, and $d(t)$ is bounded disturbance.

The control objective is to design an observer based adaptive fuzzy controller for system (1) such that the tracking error and observer error are ultimately bounded while all signals in the closed-loop system remain bounded.

In this paper, we will make the following assumptions concerning the system (1) and the desired trajectory $x_m(t)$.

Assumption 1: without loss of generality, it is assumed that the nonzero function $g_i(\underline{x}_i)$ satisfies the following condition:

$$\begin{aligned} & g(\underline{x}) \geq g_{\min} > 0 \quad \forall \underline{x} \in R^n \\ & \frac{dg(\underline{x})}{dt} \geq g_{dm} \end{aligned} \tag{2}$$

$g_{dm} \in R$ is known constant parameter and define later. Furthermore, the following controller and observer design can be reconstructed for $g_{\min} < 0$ in same way.

Assumption 2: The desired trajectory $x_m(t)$ is generated by the following desired system.

$$\begin{cases} \dot{\underline{x}}_m = A_0 \underline{x}_m + br(t) \\ y_m = C^T \underline{x}_m \end{cases} \tag{3}$$

where $r(t)$ is external desired value.

The interactions can be considered as external inputs as a function of the states of the subsystems, thus it is bounded by some constant time varying signal, which is in general a function of all the states. To make it more suitable for the

proposed controller derivation, the following assumption is used.

Assumption 3: the external disturbance satisfies the following property.

$$\|d(t)\|_{\infty} \le d_{\max} \tag{4}$$

Consider $\underline{\hat{x}}(t)$ as an estimation of $\underline{x}(t)$ and the following definitions.

$$\begin{aligned} \underline{e} &= \underline{x}_{\mathrm{m}} - \underline{x} = [e\ \dot{e} \ldots e^{(n-1)}]^{\mathrm{T}} \\ \underline{\hat{e}} &= \underline{x}_{\mathrm{m}} - \underline{\hat{x}} = [\hat{e}\ \ \dot{\hat{e}} \ldots \hat{e}^{(n-1)}]^{\mathrm{T}} \\ \underline{\tilde{e}} &= \underline{e} - \underline{\hat{e}} \end{aligned} \tag{5}$$

where $\underline{e}$ stands for the tracking error, $\underline{\hat{e}}$ presents the observer error and $\underline{\tilde{e}}$ is for the observation error.

Consider the following tracking error vector.

$$\underline{e} = [e_1, e_2, \ldots, e_n]^{\mathrm{T}} \in R^{n_i} \tag{6}$$

Taking the derivative of both sides of the equation (6) we have

$$\begin{cases} \dot{\underline{e}} = \dot{\underline{x}}_{\mathrm{m}} - \dot{\underline{x}} = A_0 \underline{x}_m + br(t) - A_0 \underline{x} - f'(\underline{x}) \\ \qquad - b\left(g(\underline{x})u + d(t)\right) \\ \underline{e}_y = C^T \underline{e} \end{cases} \tag{7}$$

Use equation (7) to rewrite the above equation as:

$$\begin{cases} \dot{\underline{e}} = A_0 \underline{e} - f'(\underline{x}) + \mathrm{b}\{r(t) - g(\underline{x})u - d(t)\} \\ \underline{e}_y = C^T \underline{e} \end{cases} \tag{8}$$

To construct the controller, let v be defined as

$$\mathrm{v} = r(t) + k_c^T \underline{\hat{e}} + v' \tag{9}$$

Consider the vector $k_c = [k_1, k_2, \ldots, k_n]^{\mathrm{T}}$ be coefficients of $\psi(s) = s^n + k_n s^{n-1} + \ldots + k_1$ and chosen so that the roots of this polynomial are located in the open left-half plane. This makes the matrix $A_{oc} = A_0 - bk_c^T$ be Hurwitz.

By adding and subtracting the term $\left(k_c^T \underline{\hat{e}} + v'\right)$ from the right-hand side of equation (9), we obtain

$$\begin{cases} \dot{\underline{e}} = A_0 \underline{e} - bk_c^T \underline{\hat{e}} - f'(\underline{x}) - b\{g(\underline{x})u \\ \qquad - v + d(t) + v'\} \\ e_y = \mathrm{C}^{\mathrm{T}} \underline{e} \end{cases} \tag{10}$$

Using assumption (1), equation (9) and the signal v which is not explicitly dependent on the control input u, the following inequality is satisfied:

$$\frac{\partial(g(\underline{x})u - v)}{\partial u} = \frac{\partial g(\underline{x})u}{\partial u} = g(\underline{x}) > 0 \tag{11}$$

Invoking the implicit function theorem, it is obvious that the nonlinear algebraic equation $g(\underline{x})u - v = 0$ is locally soluble for the input u for an arbitrary $(\underline{x}, v)$. Thus, there exists some ideal controller $u^*(\underline{x}, v)$ satisfying the following equality for a given $(\underline{x}, v) \in R^{\mathrm{n}} \times R$:

$$g(\underline{x})u^* - v = 0 \tag{12}$$

As a result of the mean value theorem, there exists a constant λ in the range of $0 < \lambda < 1$, such that the nonlinear function $g(\underline{x})u$ can be expressed around u^* as:

$$\begin{aligned} g(\underline{x})u &= g(\underline{x})u^* + (u - u^*)g(\underline{x}) \\ &= g(\underline{x})u^* + e_u g(\underline{x}) \end{aligned} \tag{13}$$

Substituting equation (13) into the error equation (10) and using (12), we get

$$\begin{cases} \dot{\underline{e}} = A_0 \underline{e} - bk_c^T \underline{\hat{e}} - f'(\underline{x}) - b\{e_u g(\underline{x}) + d(t) + v'\} \\ e_y = \mathrm{C}^{\mathrm{T}} \underline{e} \end{cases} \tag{14}$$

However, the implicit function theory only guarantees the existence of the ideal controller $u^*(\underline{x}, v)$ for system (12), and does not recommend a technique for constructing solution even if the dynamics of the system are well known. In the following, a fuzzy system and classic controller will be used to obtain the unknown ideal controller.

III. Observer Based Fuzzy Adaptive Controller Design

In the last section, it has been shown that there exists an ideal control for achieving control objectives. We show how to develop a fuzzy system to adaptively approximate the unknown ideal controller.

This section deals with the observer and controller design procedure. To design observer for the mentioned system in equation (14), this paper proposes the following observer error.

$$\begin{cases} \dot{\underline{\hat{e}}} = \underbrace{(A_0 - bk_c^T)}_{A_{oc}} \underline{\hat{e}} + \mathrm{K}_0 C^T \underline{\tilde{e}} + bk_{no}(\underline{\tilde{e}}, \underline{\hat{e}}) \left| C^{\mathrm{T}} \underline{\tilde{e}} \right| \\ \hat{e}_y = \mathrm{C}^{\mathrm{T}} \underline{\hat{e}} \end{cases} \tag{15}$$

where K_o, k_{no} are the linear observer gain and the nonlinear observer gain respectively. K_o is selected to make sure that the characteristic polynomial of $(A_{oo} = A_o - K_0 C^T)$ is Hurwitz. Defining the observation error by subtracting (14) from (15) yields

$$\begin{cases} \dot{\underline{\tilde{e}}} = \underbrace{(A_0 - K_0 C^T)}_{A_{oo}} \underline{\tilde{e}} - f'(\underline{x}) \\ \quad - b\{e_u g(\underline{x}) + d(t) + v' + k_{no}(\underline{\tilde{e}}, \underline{\hat{e}}) \left| C^{\mathrm{T}} \underline{\tilde{e}} \right|\} \\ \tilde{e}_y = C^{\mathrm{T}} \underline{\tilde{e}} \end{cases} \tag{16}$$

By using the above equation and after some manipulation, the state-space realization can be written as [10]:

$$\begin{cases} \dot{\tilde{\underline{e}}}_s = A_{oo}\tilde{\underline{e}}_s - B_s f_f'(\underline{x}) - b_s\{e_u g_f(\underline{x}) \\ \qquad + d_f(t) + v_f' + k_{nof}(\tilde{\underline{e}}, \hat{\underline{e}})\left|\mathrm{C}^{\mathrm{T}}\tilde{\underline{e}}_s\right|\} \\ \tilde{e}_y = \mathrm{C}_s^{\mathrm{T}}\tilde{e}_s \end{cases} \quad (17)$$

The ideal controller can be represented as:

$$u^* = \psi(\underline{z}) + \varepsilon_u \quad (18)$$

where $z = [x, v]^T$ and $\psi(\underline{z}) = \theta_1^* w_1(\underline{z})$, and θ_1^* and $w_1(\underline{z})$ are consequent parameters and a set of fuzzy basis functions, respectively. ε_u is an approximation error that satisfies $|\varepsilon_u| \le \varepsilon_{\max}$ and $\varepsilon_{\max} > 0$. The parameters θ_1^* are determined through the following optimization.

$$\theta_1^* = \arg\min_{\theta_1}\left[\sup\left|\theta_1^T w_1(\underline{z}) - \psi(\underline{z})\right|\right] \quad (19)$$

Denote the estimate of θ_1^* as θ_1 and u_{rob} as a robust controller to compensate approximation error, uncertainties, disturbance and interconnection term. To rewrite the controller given in (18) as:

$$u = \theta_1^T w_1(\underline{z}) + u_{rob} + \hat{\underline{e}}^T P_2 \mathrm{K}_0 \quad (20)$$

u_{irob} is defined below.

$$u_{rob} = sign(C_s^T\tilde{\underline{e}}_s)\left(\frac{N}{2f_{\min}}\left|C_s^T\tilde{\underline{e}}_s\right| + u_{com} + \frac{u_r}{f_{\min}} + \frac{\hat{v}'}{f_{\min}} + \hat{k}_{nof}(\tilde{\underline{e}}, \hat{\underline{e}})\left(\frac{\left|C_s^T\tilde{\underline{e}}_s\right|}{f_{\min}} + \left|\hat{\underline{e}}^T P_1 b\right|\right)\right) \quad (21)$$

In the above, $\theta_{i1}^T w_{i1}(\underline{z})$ approximates the ideal controller, u_{icom} compensates for approximation errors and uncertainties, u_{ir} is designed to compensate for bounded external disturbances, $\hat{k}_{inof}(\tilde{\underline{e}}_i, \hat{\underline{e}}_i)\left(\left|C_{is}^T\tilde{\underline{e}}_{is}\right| / f_{\min} + \left|\hat{\underline{e}}_i^T P_{i1} b_i\right|\right)$ tries to estimate the nonlinear gain of the observer, and $\hat{v}_i'$ is estimation of v_i'. Consider the following update laws.

$$\begin{aligned} \dot{\hat{k}}_{no} &= \gamma_{ko}\left(\frac{\left|C_s^T\tilde{\underline{e}}_s\right|^2}{g_{\min}} + \left|C_s^T\tilde{\underline{e}}_s\right|\left|b^T P_2\hat{\underline{e}}\right|\right) \\ \dot{\theta}_1 &= \Gamma_1 C_s^T\tilde{\underline{e}}_s w_1(\underline{z}) \\ \dot{u}_r &= \gamma_{u_r}\left|C_s^T\tilde{\underline{e}}_s\right| \\ \dot{u}_{com} &= \gamma_{u_{com}}\left|C_s^T\tilde{\underline{e}}_s\right| \\ \dot{\hat{v}}' &= \gamma_{\hat{v}'}\left|C_s^T\tilde{\underline{e}}_s\right| \end{aligned} \quad (22)$$

For more information about closed loop stability, refer to [9].

IV. Simulation Results

In this section, we apply the proposed fuzzy model reference adaptive controller to an inverted pendulum problem

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \left(\frac{mgr}{j}\right)\sin(x_1) + \frac{\alpha}{j}sat(u) \\ y = x_1 \end{cases} \quad (23)$$

where y are the angular displacements of the pendulums from vertical position. $m = 2\ kg$ are the pendulum end masses, j=0.5 kg is the moment of inertia, r=0.5 m is the height of the pendulum, $g = 9.81 m/s^2$ shows the gravitational acceleration, $\alpha = 25$ are the control input gain.

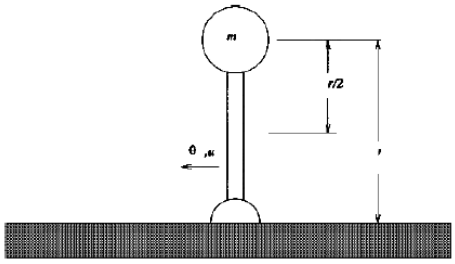
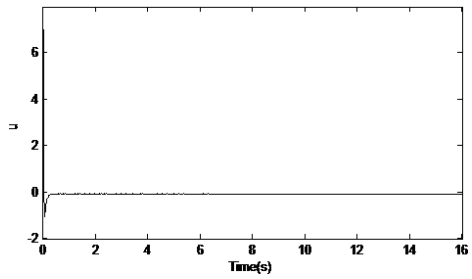
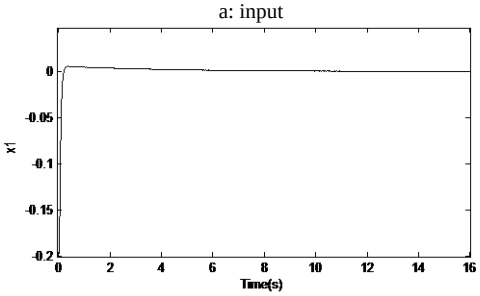


Fig. 1. Inverted pendulum



a: input



b: output of system

Fig. 2. Performance of the proposed controller

Applying the proposed controller to the above system. As shown in figure (2-a), it is obvious that the performance of the proposed controller is promising. Figure (2-b) shows the total input of system.

V. Conclusion

In this paper, we propose a new fuzzy model reference adaptive controller for affine nonlinear systems. The asymptotic convergence of the tracking error to a neighborhood of zero, the stability of closed-loop system, and Robustness against external disturbances are the merits of proposed controller.

REFERENCES

[1] G. Feng, S. G. Cao, and N. W. Rees, "Stable adaptive control of fuzzy dynamic systems," *Elsevier Science, Fuzzy Sets and Systems*, vol. 131, pp. 217, 2002.

[2] Y. C. Hsu, G. Chen, S. Tong, and H. X. Li, "Integrated fuzzy modeling and adaptive control for nonlinear systems," *Elsevier Science, Information Sciences*, vol. 153, pp. 217, 2003.

[3] C. C. Cheng and S. H. Chien, "Adaptive sliding mode controller design based on T–S fuzzy system models," *Automatica*, vol. 42, pp. 1005, 2006.

[4] C.W. Park and M. Park, "Adaptive parameter estimator based on T–S fuzzy models and its applications to indirect adaptive fuzzy control design," *Information Sciences*, vol. 159, pp. 125, 2004.

[5] Y. Tang, N. Zhang, and Y. Li, "Stable fuzzy adaptive control for a class of nonlinear systems," *Fuzzy Sets and Systems*, vol. 104, pp. 279, 1999.

[6] L. Zhang, "Stable Fuzzy Adaptive Control Based on Optimal Fuzzy Reasoning," *IEEE*, in *Proceedings of the Sixth International Conference on Intelligent Systems Design and Applications* (*ISDA*), 2006.

[7] H. F. Ho, Y. K. Wong , A. B. Rad, and W. L. Lo, "State observer based indirect adaptive fuzzy tracking control," *Simulation Modeling Practice and Theory*, vol. 13, pp. 646, 2005.

[8] C. C. Chiang, "Adaptive Fuzzy Sliding Mode Control For Time-Delay Uncertain Large-Scale Systems," in *Proceedings of the 44th IEEE Conference on Decision and Control, and the European Control Conference*, pp. 4077, 2005.

[9] R. Ghasemi, M. B. Menhaj, and A. Afshar, "A decentralized stable fuzzy adaptive controller for large scale nonlinear systems," *Journal of Applied Science*, vol. 9, pp. 892, 2009.

[10] R. Ghasemi, M. B. Menhaj, and A. Afshar, "A New Decentralized Fuzzy Model Reference Adaptive Controller for a Class of Large-scale Non-affine Nonlinear Systems," *European Journal of Control*, vol. 15, pp. 534, 2009.

Reza Ghasemi was born in Tehran, Iran in 1979. He received his B.Sc. degrees in Electrical engineering from Semnan University in 2000 and M.Sc. degrees and Ph.D. in control engineering from Amirkabir University of Technology, Tehran, Iran, in 2004 and 2009.

His research interests include large-Scale Systems, Adaptive Control, Robust Control, Nonlinear Control, and Intelligent Systems.

Dr. Reza Ghasemi joined Islamic Azad University, Damavand Branch, the Department of Electrical Engineering, Damavand, Tehran, Iran, where he is currently an Assistant Professor of electrical engineering.

Babak Abdi was born in Tehran, in 1976. He received his MS and Ph.D. degree in electrical engineering in 2005 and 2009 from Amirkabir University of Technology (Tehran Polytechnic), Tehran, Iran, respectively. He is currently a member of IEEE and faculty member of Islamic Azad University-Damavand branch, Tehran, Iran. His research interests include power electronics, application of reliability in power electronics, Electromagnetic Interferences (EMI), electrical machines and drives.

Sayyed Mohammad Mehdi Mirtalaei was born in Shahreza-Isfahan, Iran, in 1983. He received his B.S. degree in electrical engineering from Isfan University of Technology, Iran, in 2005. he received his MS and Ph.D. in electrical engineering from Amirkabir University of Technology, Tehran, Iran, in 2007 and 2012 respectively. His research interest is power electronics, EMI/EMC and numerical method in electromagnetic.