Decentralized Intelligent Adaptive Controller for Large Scale System

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***Abstract*—A new decentralized fuzzy adaptive controller for a class of large scale affine nonlinear systems is presented in this paper. The proposed controllers are mainly based on fuzzy concepts. Stability of closed-loop system, convergence of the tracking error to zero and avoidance of chattering in adaptation laws are guaranteed in this controller design procedure. Simulation results have very promising performance.**

***Index Terms*—Intelligent adaptive control, non-affine nonlinear systems, large scale system, fuzzy system.**

I. Introduction

Nowadays, fuzzy adaptive controller (FAC) has attracted many researchers to developed appropriate controllers for nonlinear systems especially for large scale systems (LSS) due to its tunable structure, the performance or the FAC is superior that of the fuzzy controller. Further, instead of using adaptive controller, FAC can use knowledge of the experts in the controller.

In the recent year, FAC has been fully studied. The Takagi-Sugeno (TS) fuzzy systems have been used to model nonlinear systems and then TS based controllers have been designed with guaranteed stability [1], [2]. To model affine nonlinear system and to design stable TS based controllers have been employed in [3]. Designing of the sliding mode fuzzy adaptive controller for a class of multivariable TS fuzzy systems are presents in [4]. In [5], [6], the non-affine nonlinear function are first approximated by the TS fuzzy systems, and then stable TS fuzzy controller and observer are designed for the obtained model. In these papers, modeling and controller has been designed simply, but the systems must be linearizable around some operating points.

[7] have considered linguistic fuzzy systems to design stable adaptive controller for affine systems based on feedback linearization. Stable FAC based on sliding mode is designed for affine systems in [8]. Designing fuzzy adaptive output Tracking Controller for a class of Non-affine Nonlinear Systems with nonlinear output mapping is proposed in [9]. Designing decentralized fuzzy adaptive controller for a class of large scale system is discussed in [10], [11]. Adaptive State Tracking Controller for Multi-Input Multi-Output Non-affine Nonlinear Systems is presented in [12].

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II. Problem Statement

Consider the following large scale affine nonlinear system.

$$\begin{cases} \dot{x}_{i,l} = x_{i,l+1} \\ \dot{x}_{i,n_i} = f_i(X_i) + g_i(X_i)u_i \\ +\Delta_i(X_1, X_2, ..., X_N) + d_i(t) \\ y_i = x_{i,1} \quad i = 1, 2, ..., N \end{cases} \tag{1}$$

$$l = 1, 2, ..., n_i - 1$$

where x_{ij} is jth state of ith subsystem, $X_i = [x_{i,1}, \ldots, x_{i,n_i}]^T \in \mathbb{R}^{n_i}$ is the state vector of the ith subsystem which is assumed available for measurement, $u_i \in R$ is the control input, $y_i \in R$ is the system output, $f_i(X_i)$, $g_i(X_i)$ are unknown smooth nonlinear function, $\Delta_i(X_1, X_2, ..., X_N)$ is an unknown nonlinear interconnection term, and $d_i(t)$ is bounded disturbance.

The control objective is to design an adaptive fuzzy controller for system (1) such that the system output $y_i(t)$ follows a desired trajectory $y_d(t)$ while all signals in the closed-loop system remain bounded.

In this paper, we will make the following assumptions concerning the system (1) and the desired trajectory $y_d(t)$.

The error of the system can be rewritten as:

$$\begin{aligned} \dot{e}_i = A_{i0}e_i + b_i\{y_d^{(n_i)} - f_i(X_i, u_i) \\ -\Delta_i(X_1, X_2, ..., X_N) - d_i(t)\} \end{aligned} \tag{2}$$

where A_{i0} and b_i are defined below.

$$A_{i0} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \in R^{n_i \times n_i} \tag{3}$$

$$b_i = \begin{bmatrix} 0 & 0 & \cdots & 1 \end{bmatrix}^T \in R^{n_i}$$

Consider the vector $k_i = [k_{i,1}, k_{i,2}, \ldots, k_{i,n_i}]^T$ be coefficients of $L(s) = s^{n_i} + k_{i,n_i}s^{n_i-1} + ... + k_{i,1}$ and chosen so that the roots of this polynomial are located in the open left-half plane. This makes the matrix $A_i = A_{i0} - b_i k_i^T$ be

Hurwitz. Thus, for any given positive definite symmetric matrix Q_i, there exists a unique positive definite symmetric solution P_i for the following Lyapunov equation:

$$A_i^T P_i + P_i A_i = -Q_i \tag{4}$$

Let v_i be defined as

$$v_i = y_d^{(n_i)} + k_i^T e_i + \beta \tanh(b_i^T P_i e_i / \varepsilon) + v_i' \tag{5}$$

where $\tanh(b_i^T P_i e_i / \varepsilon)$ is the hyperbolic tangent function, β is a large positive constant, and ε is a small positive constant.

By adding and subtracting the term $k_i^T e_i + \beta \tanh(b_i^T P_i e_i / \varepsilon) + v_i'$ from the right-hand side of equation (2), we obtain

$$\begin{aligned} \dot{e}_i = {} & A_i e_i - b_i \{ f_i(X_i, u_i) - v_i \\ & + \Delta_i(X_1, X_2, ..., X_N) + d_i(t) \\ & + \beta \tanh(b_i^T P_i e_i / \varepsilon) + v_i' \} \end{aligned} \tag{6}$$

Using assumption (1), equation (5) and the signal v_i which is not explicitly dependent on the control input u_i, the following inequality is satisfied:

$$\frac{\partial (f_i(X_i) + g_i(X_i) u_i) - v_i)}{\partial u_i} = g_i(X_i) > 0 \tag{7}$$

Invoking the implicit function theorem, it is obvious that the nonlinear algebraic equation $f_i(X_i) + g_i(X_i) u_i - v_i = 0$ is locally soluble for the input u_i for an arbitrary (X_i, v_i). Thus, there exists some ideal controller $u_i^*(X_i, v_i)$ satisfying the following equality for a given $(X_i, v_i) \in R^{n_i} \times R$:

$$f_i(X_i) + g_i(X_i) u_i^* - v_i = 0 \tag{8}$$

As a result of the mean value theorem, there exists a constant λ in the range of $0 < \lambda < 1$, such that the nonlinear function $f_i(X_i, u_i)$ can be expressed around u_i^* as:

$$\begin{aligned} & f_i(X_i) + g_i(X_i) u_i \\ & = f_i(X_i) + g_i(X_i) u_i^* + (u_i - u_i^*) f_{iu_\lambda} \\ & = f_i(X_i) + g_i(X_i) u_i^* + e_{u_i} f_{iu_\lambda} \end{aligned} \tag{9}$$

where $f_{iu_\lambda} = \partial f(X_i, u_i) / \partial u_i |_{u_i = u_{i\lambda}}$ and $u_{i\lambda} = \lambda u_i + (1 - \lambda) u_i^*$.

Substituting equation (9) into the error equation (6) and using (8), we get

$$\begin{aligned} \dot{e}_i = {} & A_i e_i - b_i \{ e_{u_i} f_{iu_\lambda} + \Delta_i(X_1, X_2, ..., X_N) \\ & + d_i(t) + \beta \tanh(b_i^T P_i e_i / \varepsilon) + v_i' \} \end{aligned} \tag{10}$$

However, the implicit function theory only guarantees the existence of the ideal controller $u_i^*(X_i, v_i)$ for system (8), and does not recommend a technique for constructing solution even if the dynamics of the system are well known. In the following, a fuzzy system and classic controller will be used to obtain the unknown ideal controller.

III. Fuzzy Adaptive Controller Design

In previous section, it has been shown that there exists an ideal control for achieving control objectives. In this section, we show how to develop a fuzzy system to adaptively approximate the unknown ideal controller.

The ideal controller can be represented as:

$$u_i^* = f_i(z) + \varepsilon_{iu} \tag{11}$$

where $f_i(z) = \theta_{i1}^* w_{i1}(z)$, and θ_{i1}^* and $w_{i1}(z)$ are consequent parameters and a set of fuzzy basis functions, respectively. ε_{iu} is an approximation error that satisfies $|\varepsilon_{iu}| \le \varepsilon_{\max}$ and $\varepsilon_{\max} > 0$. The parameters θ_{i1}^* are determined through the following optimization.

$$\theta_{i1}^* = \arg\min_{\theta_{i1}} \left[\sup \left| \theta_{i1}^T w_{i1}(z) - f_i(z) \right| \right] \tag{12}$$

Denote the estimate of θ_{i1}^* as θ_{i1} and u_{irob} as a robust controller to compensate approximation error, uncertainties, disturbance and interconnection term to rewrite the controller given in (17) as:

$$u_i = \theta_{i1}^T w_{i1}(z) + u_{irob} \tag{13}$$

In which u_{irob} is defined below.

$$\begin{aligned} u_{irob} = {} & \frac{|b_i^T P_i e_i|}{f_{\min} b_i^T P_i e_i} \Big(\frac{1}{2} \sum_{j=1}^N \hat{\theta}_{ji2}^T w_{i2}(|b_i^T P_i e_i|) |b_i^T P_i e_i| \\ & + \hat{\psi}_{i0} + \frac{N}{2} |b_i^T P_i e_i| u_{ic} + f_{\min} u_{icom} + f_{\min} u_{ir} + \hat{v}_i' \Big) \end{aligned} \tag{14}$$

In the above, $\theta_{i1}^T w_{i1}(z)$ approximates the ideal controller, $\hat{\psi}_{i0} + \frac{1}{2} \sum_{j=1}^N \hat{\theta}_{ji2}^T w_{i2}(|b_i^T P_i e_i|) |b_i^T P_i e_i|$ tries to estimate the interconnection term, u_{icom}, u_{ic} compensate for approximation errors and uncertainties, u_{ir} is designed to compensate for bounded external disturbances, and $\hat{v}_i'$ is estimation of v_i'.

Consider the following update laws.

$$
\begin{aligned}
\dot{\theta}_{i1} &= \Gamma_1 \vartheta^{-1}(t) b_i^T P_i e_i w_{i1}(z_i) \\
\dot{\hat{\psi}}_{i0} &= \frac{\gamma_{\psi_{i0}}}{f_{\min}} \vartheta^{-1}(t) \left| b_i^T P_i e_i \right|, \\
\dot{\theta}_{ij2} &= \frac{\Gamma_{j2}}{2 f_{\min}} \vartheta^{-1}(t) \left| b_i^T P_i e_i \right|^2 w_{i2}(\left| b_i^T P_i e_i \right|) \\
\dot{u}_{ir} &= \gamma_{u_{ir}} \vartheta^{-1}(t) \left| b_i^T P_i e_i \right| \\
\dot{u}_{icom} &= \gamma_{u_{icom}} \vartheta^{-1}(t) \left| b_i^T P_i e_i \right| \\
\dot{u}_{ic} &= \frac{\gamma_{u_{ic}} N}{2 f_{\min}} \vartheta^{-1}(t) \left| b_i^T P_i e_i \right|^2 \\
\dot{\hat{v}}_i' &= \frac{\gamma_{\hat{v}_i'}}{f_{\min}} \vartheta^{-1}(t) \left| b_i^T P_i e_i \right|
\end{aligned}
\tag{15}
$$

where $\Gamma_1 = \Gamma_1^T > 0$, $\Gamma_{j2} = \Gamma_{j2}^T > 0$, $, \gamma_{u_{ir}}, \gamma_{u_{icom}}, \gamma_{u_{ic}}, \gamma_{\hat{v}_i'} > 0$ are constant parameters.

Theorem 2: consider the error dynamical system given in (10) for the large scale system (1), interconnection term satisfying assumption (3), the external disturbances and a desired trajectory, then the controller structure given in (13), (14) with adaptation laws (15) makes the tracking error and error of parameters estimation converge asymptotically to a neighborhood of origin.

Proof: refer to [10].

IV. Simulation Results

In this section, we apply the proposed decentralized fuzzy model reference adaptive controller to a two-inverted pendulum problem [12] in which the pendulums are connected by a spring as shown in figure (1). The pendulums dynamics are described by the following nonlinear equations.

$$
\begin{cases}
x_{11} = x_{12} \\
x_{12} = \left(\frac{m_1 g r}{j_1} - \frac{k r^2}{4 j_1}\right) \sin(x_{11}) + \frac{k r}{2 j_1}(l - b) \\
\qquad + \frac{\alpha}{j_1} u_1 + \frac{k r^2}{j_1} \sin(x_{21}) + d(t) \\
y_1 = x_{11}
\end{cases}
\tag{16}
$$

$$
\begin{cases}
x_{21} = x_{22} \\
x_{22} = \left(\frac{m_2 g r}{j_2} - \frac{k r^2}{4 j_2}\right) \sin(x_{21}) + \frac{k r}{2 j_2}(l - b) \\
\qquad + \frac{\alpha}{j_2} u_2 + \frac{k r^2}{j_2} \sin(x_{12}) + d(t) \\
y_2 = x_{21}
\end{cases}
$$

where y_1, y_2 are the angular displacements of the pendulums from vertical position. $m_1 = 2kg$, $m_2 = 2.5kg$ are the pendulum end masses, $j_1 = 0.5kg$, $j_2 = 0.62kg$ are the moment of inertia, $k = 100\, N/m$ is spring constant, $r = 0.5m$ is the height of the pendulum, $g = 9.81\, m/s^2$ shows the gravitational acceleration, $l = 0.5m$ is the natural length of spring, $\alpha_1, \alpha_2 = 25$ are the control input gains and $b = 0.4m$ presents distance between the pendulum hinges. Furthermore it is assumed $d(t) = \sin(200\pi t)$.

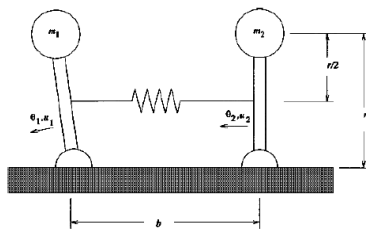
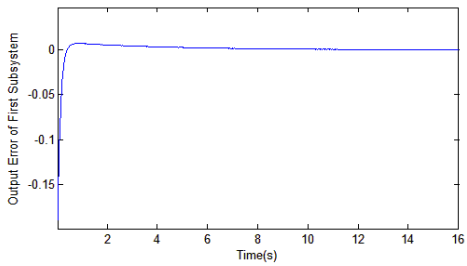
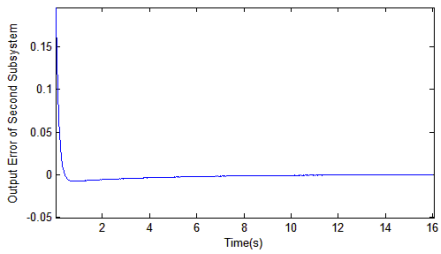


Fig. 1. Two inverted pendulum connected by a spring

It is clear the states of system x_{i1}, x_{i2} in the range of $[-1,1],[-5,5]$. Let $X_i = [x_{i1}, x_{i2}]^T$, $z_i = [x_{i1}, x_{i2}, v_i]^T$ and v_i are defined over $[-45,45]$. For each fuzzy system input, we define 6 membership functions over the defined sets. Consider that all of the membership functions are defined by the Gaussian function $\mu_j(\chi) = \exp\left(\frac{(\chi - c)^2}{2\delta^2}\right)$, where c is center of the membership function and δ is its variance. We assume that the initial value of $\theta_{i1}(0)$, $\theta_{i2}(0)$, $u_{ir}(0)$, $u_{icom}(0)$, and $\hat{v}_i'(0)$ be zero.

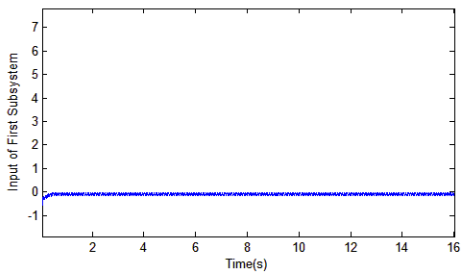


a: in first subsystem

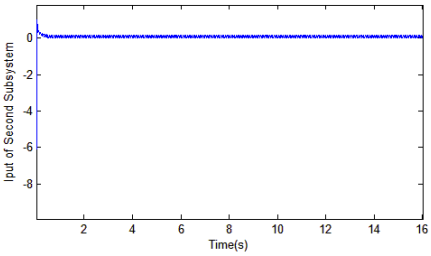


b: in second subsystem

Fig. 2. Performance of the proposed controller



a: u1



b: u2

Fig. 3. Control input

Furthermore, it has been assumed that $f_{\min} = 1$, $\Gamma_1 = 10$, $\Gamma_2 = 10$, $\gamma_{u_{com}} = 5$, $\gamma_{u_r} = 5$, $\gamma_{\hat{v}_i'} = 5$. In equation (18) and remark (1), we assume that $\sigma = 0.01$, $\varepsilon = 0.01$. The parameters $f_{dm}, f_{\min}$ and the vector $k_i = [k_{i,1}, k_{i,2}, \ldots, k_{i,n_i}]^T$ has been chosen so that the lemma 2 holds. As shown in figures (2-a and b), it is obvious that the performance of the proposed controller is promising. Figures (3-a and b) shown the total input of each subsystem.

V. Conclusion

Developed a new method for designing a decentralized adaptive controller using fuzzy systems for a class of large-scale nonlinear non-affine systems with unknown nonlinear interconnections is discussed in this paper. The properties of the proposed adaptive controller are as: 1) stability of closed-loop, 2) convergence of the tracking errors to zero 3) Robustness against external disturbances.

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