Impulse Noise Removal from Images: A Wavelet Network Approach

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Abstract—This paper presents a new two-stage approach to impulse noise removal for images based on wavelet network (WN). The first step of this two-stage approach is noise detection, in which the so-called Gray-Level Difference (GD) and Average Background Difference (ABD) is considered as the input of a Wavelet Network (WN). The second step is removing impulse noise with a median filter. The Wavelet Network presented here is a Fixed Grid Wavelet Network (FGWN) without learning. Experimental results of the study show that our method acts on impulse noise effectively, and at the same time preserves chromaticity and image details very well.

Index Terms—Impulse noise, wavelet network, gray-level difference, average background difference

I. INTRODUCTION

Impulse noise is frequently encountered in acquisition, transmission and processing of images. The present of impulse noise in an image maybe either relatively high or low. Thus, it could severely degrade the image quality and cause some loss of image information details. Filtering a digital image to remove noise while keeping the image details is an essential part of image task. Various filtering techniques have been proposed for removing impulse noise in the literature and it is well-known that linear filters could produce serious image blurring. As a result, nonlinear filters have been widely exploited due to their improved filtering performance. Vector Median Filter (VMF) provides efficient impulse noise attenuation. However, it treats all pixels of a color image in the same way and also modifies pixels that are not corrupted by noise. To avoid the damage of uncorrupted pixels, two-step filtering methods (e.g. [1]) can be used. The idea is based on impulse noise detection and noise removal.

Wavelet network (WN) is a new network based on wavelet transforms [2], [3] in which discrete wavelet function is used as node activation function. In fact the wavelet space is used as characteristic space of the pattern recognition task. Furthermore, WN combines the advantages of time-frequency localization wavelet transform with the classifying power of neural network. This increases accuracy and robustness. The main advantage of wavelet networks over similar architectures such as multi-layer perceptrons and networks of radial basis functions (RBF) is the possibility of optimizing the wavelet network structure by means of efficient deterministic construction algorithms. However, owing to the localized nature of the wavelet basis functions, wavelet networks may not be well-suited to

dealing with high-dimensional data.

In this paper a new impulse noise removal technique based on wavelet network is proposed to restore digital images corrupted by impulse noise. First, wavelet network is employed to detect the noisy-pixels and distinguish it from noise-free pixels. The WN inputs are two parameters; Gray-Level Difference (GD) and Average Background Difference (ABD); that have been calculated before. Second, the median filter improves the noisy pixels. A series of experimental results show that the proposed method unlike conventional filters is capable of effectively eliminating the impulse noise while preserving more fine details. The wavelet network proposed here is a fixed grid wavelet network in which translation and dilation parameters are already determined and only the weight of network is optimized by training the network. In general, gradient type algorithms are not needed to train such a network. Here Orthogonal Least Square (OLS) algorithm is used to determine the network parameters. In comparison with back propagation (BP), OLS is a much faster approach.

This paper is organized as follows. The basic concepts of two type's wavelet network are introduced in section II. In this section, the WN structure with two algorithms necessary for formation of network and determination its parameters are presented. In section III, Impulse noise model and an algorithm for detecting and cancelling it based on wavelet network will be introduced. The simulation results of the proposed method in comparison with other approaches will be illustrated in section IV. Finally, in section V a summary and conclusion of the findings of the study will be provided.

II. WAVELET NETWORK

A wavelet network is a nonlinear regression structure that implements input-output mappings as the superposition of dilated and translated versions of a mother wavelet function, which is localized both in the space and frequency domain [4]. Such a structure can approximate any square-integrable function to an arbitrary precision, given a sufficiently large number of network elements. Both continuous and discrete wavelet transforms have been introduced to implement neural networks.

Existing WNs can, therefore, be classified into two types [5]. First, Adaptive WNs (AWN), where wavelets as activation functions stem from the continuous wavelet transform and the unknown parameters of the networks include the weighting coefficients and the dilation and translation factors of the wavelets. These parameters can be viewed as coefficients varying continuously as in conventional neural networks and can be learned by gradient type algorithms. Second, Fixed grid WNs (FGWN), where

the activation functions stem from the discrete wavelet transforms and unlike in adaptive neural networks, the unknown inner parameters of the networks vary on some fixed discrete lattices. In such a WN, the positions and dilations of the wavelets are fixed (predetermined) and only the weights have to be optimized by training the network. In general, gradient type algorithms are not needed to train such a network. An alternative solution for training this type of network is to convert the networks into a linear-in-the-parameter problem, which can then be solved using least squares type algorithms.

AWN has many drawbacks such as; determination initial value of parameters and complicated calculations especially for high dimensional models. For this reason, in this study FGWN is employed instead of AWN.

Fig. 1. Wavelet network structure

A. Structure of Wavelet Network

The output signal of a wavelet network with one output y , d inputs $\{x_1, x_2, \dots, x_d\}$ and q nodes in the hidden layer is $y = \sum_{i=1}^q \omega_i \psi_{a_i, t_i}(\mathbf{x})$, where $\mathbf{x} = (x_1, x_2, \dots, x_d)^T$ is the vector of inputs, $\omega_i, i = 1, 2, \dots, q$ are weight coefficients between hidden and output layers, and ψ_{a_i, t_i} are dilated and translated versions of a mother wavelet function $\psi: \mathbb{R}^d \rightarrow \mathbb{R}: \psi_{a_i, t_i}(\mathbf{x}) = a_i^{-d/2} \psi(\frac{\mathbf{x} - \mathbf{t}_i}{a_i})$.

Function ψ , termed the “mother wavelet”, is required to have zero mean and also to be localized both in the space and frequency domains [2]. A number of methods are available to construct multidimensional mother wavelets (i.e., with $d > 1$) from one-dimensional functions $\psi: \mathbb{R} \rightarrow \mathbb{R}$ with fast decay in space and frequency [4]. In the present study, the radial wavelets are used to implement WNs. For example the d -dimensional Mexican hat wavelet can be expressed as $\psi(\mathbf{x}) = \phi(\|\mathbf{x}\|) = (d - \|\mathbf{x}\|^2) \exp(-\|\mathbf{x}\|^2/2)$.

According to above definitions, pairs (a_i, t_i) are taken from a grid Λ given by $\Lambda = \{(\alpha^{-m}, \beta \mathbf{n} \alpha^{-m}) : m \in \mathbb{Z}, \mathbf{n} \in \mathbb{Z}^d\}$, where the scalar parameters α and β define the step sizes of dilation and translation discretization, respectively.

Typical choices for α , β are $\alpha = 2$ and $\beta = 1$ which are adopted throughout this work. So $(a_i, t_i) = (2^{-m_i}, \mathbf{n}_i 2^{-m_i})$ and then the output of WN is given

by

$$y = \sum_{i=1}^q \omega_i \psi_{m_i, \mathbf{n}_i}(\mathbf{x}) = \sum_{i=1}^q \omega_i 2^{-m_i d/2} \psi_{m_i, \mathbf{n}_i}(2^{m_i} \mathbf{x} - \mathbf{n}_i) \quad (1)$$

Any function $f \in L^2(\mathbb{R}^d)$ (finite-energy and continuous or discontinuous) can be approximated by an arbitrary precision using the wavelet network given in Fig.1 with Eq. (1).

B. Building a Wavelet Network

A major advantage of wavelet networks over other neural architectures is the availability of efficient construction algorithms for defining the network structure. After the structure has been determined, the weights ω_i in (1) can be obtained through linear estimation techniques. In this work, a constructive method similar to that introduced in [3] and [4]. Is employed to build a wavelet network. It can be described as follows:

Suppose that M modeling samples are available in the form of input-output pairs $(\mathbf{x}[k], y[k])$, $k = 1, 2, \dots, M$, where $(\mathbf{x}[k])_{d \times 1}$ is a column vector. Then

- 1) Normalize the input data to fit within the effective support H of the mother wavelet employed. For radial wavelets, H is a hypersphere in $\mathbb{R}^d$ with radius R . For computational simplicity, H is approximated as a hypercube inscribed in the hypersphere with edges parallel to the coordinate axis.
- 2) Choose m_{min} and m_{max} the minimum and maximum scale levels to be employed.
- 3) For each sample $\mathbf{x}[k]$ in the modeling set, find I_k the index set of wavelets whose effective supports contain $\mathbf{x}[k]$:

$$I_k = \{(m, \mathbf{n}) : \mathbf{x}[k] \in H_{m, \mathbf{n}}; m_{min} \leq m \leq m_{max}\}$$
, where $H_{m, \mathbf{n}}$ is a hypercube centered at $\mathbf{n} 2^m$ with edges $2^m R \sqrt{2}$.
- 4) Determine the pairs $(m, \mathbf{n})$ which appear in at least two sets I_{k_1} and I_{k_2} , $k_1 \neq k_2$. These are the wavelets whose effective supports include at least two samples. This step is different from the algorithm described in [3], which allows for wavelets with effective supports containing only one sample. Such wavelets are not included here because they would introduce oscillations between neighbor modeling points, which might compromise the generalization ability of the model.
- 5) Let L be the number of wavelets obtained in the previous step. For notational simplicity replace the double index $(m, \mathbf{n})$ by a single index $i = 1, \dots, L$. Apply the L wavelets to the M modeling samples and gather the results in a matrix form as follows:

$$W_{M \times L} = \begin{bmatrix} \psi_1(\mathbf{x}[1]) & \psi_2(\mathbf{x}[1]) & \dots & \psi_L(\mathbf{x}[1]) \\ \psi_1(\mathbf{x}[2]) & \psi_2(\mathbf{x}[2]) & \dots & \psi_L(\mathbf{x}[2]) \\ \vdots & \vdots & \dots & \vdots \\ \psi_1(\mathbf{x}[M]) & \psi_2(\mathbf{x}[M]) & \dots & \psi_L(\mathbf{x}[M]) \end{bmatrix}$$

After these stages the WN can then be convert into form $y = \sum_{i=1}^L \omega_i \psi_i(\mathbf{x}) = W \cdot \boldsymbol{\theta}$, where $\boldsymbol{\theta}_{L \times 1} = [\omega_1 \dots \omega_L]^T$.

It is often the case that Steps 1 to 4 of the construction process results in a large number of wavelets. In order to avoid over fitting problems that result from an over parameterization of the model, it is then important to select a reduced subset of wavelets. A fast and efficient model structure determination approach has been implemented using the OLS algorithm and. This approach has been extensively studied and widely applied in nonlinear system identification [6].

According to OLS algorithm assume that we want to select the best subset of W and the size of this subset is known and denoted as s . The algorithm first selects the wavelet in W that best fits the observed data, then repeatedly selects the wavelet in the remainder of W that best fits the data while combining with the previously selected wavelets. The iterative procedure is described in [3]. After employed OLS algorithm the WN is constructed as

$$y = \sum_{i=1}^s \omega_i \psi_i(\mathbf{x}), \quad (2)$$

where s is the number of neuron in hidden layer and ω_i is weight of neurons.

III. IMPULSE NOISE REMOVAL

A. Impulse Noise Model

Impulse noise is when the pixels are randomly misfired and replaced by other values in an image. The image model containing impulse noise can be described as follows

$$X_{i,j} = \begin{cases} N_{i,j} & \text{with probability } p \\ S_{i,j} & \text{with probability } 1-p \end{cases} \quad (3)$$

where S_{ij} denotes the noiseless image pixel and N_{ij} denotes the noise replaced instead of original pixel. Parameter p shows the rate by which the image is corrupted by impulse noise. In a variety of impulse noise models for images, fixed-and random valued impulse noises are mostly discussed. Fixed-valued impulse noise, known as the salt-and-pepper noise, is made up of corrupted pixels whose values are replaced with values equal to the maximum or minimum (255 or 0) of the allowable range [1]. This paper, deals with the fixed impulse noise.

B. Noise Detection Algorithm

Since the residual noise will strongly affect human perception, precise noise detection is essential step for the noise removal. In this paper, a WN is proposed for noise detection. The input layer of WN consists of two nodes corresponding to the gray-level difference (GD) and average background difference (ABD) are assigned to each R,G, and B matrices (for color images) in the 3×3 sliding window. The second layer is the hidden layer that consists of ten nodes (in our experiments). The output layer includes one node that

represents the identified attribution of the pixel: noisy or not-noisy. The two features in the input layer are discussed in [1]. The GD represents the accumulated variations between the central pixel and its neighbor local pixels and the ABD represents the overall average variation with the central pixel in the block. These parameters are defined as

$$\begin{aligned} GD &= \sum_{i=-1}^1 \sum_{j=-1}^1 |P(0,0) - P(i,j)|, \\ ABD &= |P(0,0) - \frac{\sum_{i=-1}^1 \sum_{j=-1}^1 P(i,j)}{8}| \end{aligned} \quad (4)$$

where $P(0,0)$ is the reference pixel and $P(i,j)$ is the surrounding local pixel. According to [1] GD is a measure of noise in the flat area. It is expected that the corrupted pixels would yield much bigger differences as compared with the uncorrupted pixels. However, the pixels on edges and or texture areas will also get high GD values so that the obscure region identification between the noise, edge, and texture pixels should be relied on the other assistant features, such as ABD. In order to train the proposed WN for noise detection, the 512×512 of the Lena color image with (20%) salt and pepper noise is used as a reference pattern for training. Also, 500 noisy pixels and 500 uncorrupted pixels uniformly distributed in the image are adopted as the training data. If (i,j) the pixel is the noisy pixel the desired output is 1 ($y(i,j) = 1$) and If the (i,j) pixel is the clean pixel the desired output is -1 ($y(i,j) = -1$). In our experiments, ten nodes in the hidden layer are enough.

C. Noise Removal Algorithm

After the first level, 3×3 median filter is applied for processing. WN recognizes the pixels of an image as noisy or clean, then if the pixel is noisy its value will be replaced by its median value and if it is not noisy its value will remain unchanged. This is realized as follows

$$f(i,j) = \begin{cases} \text{median}f(i,j) & \text{if } y(i,j) = 1 \\ f(i,j) & \text{if } y(i,j) = -1 \end{cases} \quad (5)$$

The proposed two-level noise removal algorithm is very effective to suppress the impulse noise as well as to preserve the sharpness of edges and color detail information.

IV. EXPERIMENTAL RESULTS

In this section, the quantificational calculation and vision comparison are used to assess the performance of the proposed impulse noise removal technique. The signal noise ratio (SNR) and peak signal noise ratio (PSNR) are.

$$\begin{aligned} SNR &= 10 \log \frac{\sum_{i=1}^M \sum_{j=1}^N (f(i,j) - \bar{f}(i,j))^2}{\sum_{i=1}^M \sum_{j=1}^N (f(i,j) - \hat{f}(i,j))^2}, \\ PSNR &= 10 \log \frac{255^2}{\frac{1}{MN} \sum_{i=1}^M \sum_{j=1}^N (f(i,j) - \hat{f}(i,j))^2} \end{aligned} \quad (6)$$

where MN is the size of the image, f is the noisy image, $\hat{f}$ is restored the image and $\bar{f} = \frac{1}{MN} \sum_{i=1}^M \sum_{j=1}^N (f(i, j))^2$ is the mean of the corrupted image.

Fig. 3. Contrast of filters output. (a) original image; (b) the image corrupted by impulse noise (the noise density to 20%); (c) reconstructed by Vector Median Filtering; (d) reconstructed by Neural Network; (e) reconstructed by Wavelet Neural Network5; (f) reconstructed by the proposed filter.

Fig. 2. Denoising results (PSNR).

In Table I (a), the PSNR of the proposed method is

compared with that of many other well-known algorithms. The test image is color Lena, which is corrupted by 20% impulse noise. The results show that our denoising method performs other methods. In addition compared to the other methods, it can efficient preserve image details. In Table 1(b), the comparison about SNR indicates for different noise density (ND) for pepper image test is shown. A comparison between proposed WN denoising, Vector median filter (VMF), Neural Network (NN) and Wavelet Neural Network (WNN) proposed in [7] for color Lena test image was shown in Fig.2 with noisy images (NI) as an addition and the results of this comparison for noise density 20% are presented in Fig.3.

TABLE I: COMPARISON OF RESTORATION RESULT IN. (A) PSNR (DB); (B) SNR (DB) FOR DIFFERENT IMPULSE NOISE.

Algorithm	PSNR
Noisy Image	25.36
Vector median filter	28.75
Neural Network	31.63
Wavelet Neural Network [7]	32.31
Wavelet Network (proposed)	33.45

ND	NI	VMF	NN	WNN [7]	WN
10%	10.27	11.23	12.55	12.95	13.25
20%	8.11	9.02	9.16	10.25	11.23
50%	5.43	6.65	6.71	7.07	7.89
65%	4.41	5.05	5.91	6.07	6.89
80%	3.82	4.51	4.93	5.64	6.01

V. CONCLUSION

In this paper, a new efficient impulse noise removal based on WN is presented. The advantages of the WN, together with two powerful features GD and ABD are combined to remove the impulse noise. Extensive computer simulations indicate that the propose filter can suppress impulse noise and simultaneously preserve more fine details which outperforms significantly many other well-known algorithms.

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