A Consistent Binormalized Data-Reusing LMS Algorithm for Noisy FIR Models

Byung Hoon Kang, Nam Kyu Kwon, Hyon-Taek Choi, and Poo Gyeon Park

Abstract—This paper proposes a consistent binormalized data-reusing least mean square (LMS) algorithm for identifying finite impulse response models whose input and output are corrupted by additive white noise. The proposed algorithm exploits the stochastic properties of the noisy input to compensate a bias of estimation which is occurred by input noise. Furthermore, by reusing the input signal, the algorithm overcomes a decline of convergence performance with highly correlated input signal. The experimental results show that the proposed algorithm achieves consistent estimation with noisy input signal. Furthermore, the proposed algorithm gets faster convergence rate and smaller steady-state estimation errors than the ordinary consistent LMS algorithms when the input signal is highly correlated.

Index Terms—Normalized least mean square algorithm (nLMS), consistent estimation, input noise, bias compensation, FIR channel estimation.

I. INTRODUCTION

It is well known that the ordinary least squares method yields unbiased parameters estimates when the data matrix is known exactly but if the data matrix or input signal is corrupted by noise then the estimates are biased [1], [2], [3]. There are several approaches that to obtain an unbiased estimate including the total least squares method which uses the singular value decomposition of a matrix, and modified least squares method which is to remove the noise-induced bias [2], [3], [4]. However, these approaches need huge computational complexity, generally $O(N^3)$. Therefore, the algorithm which can achieve unbiased estimation while it needs less computational complexity have been studied [5],

[6]. Recently, the least mean square (LMS) type algorithm getting consistent estimate have been proposed [7]. The LMS-type algorithm can obtain consistent solution with much less computation, but has relatively slow convergence rate, especially when correlated input signal is used.

By the way, it is reported that the convergence performance can be improved by reusing input data [1].

Particularly a binormalized data-reusing LMS (BNDR-LMS) algorithm achieved good convergence rate with small additional computation [8], [9].

Inspired by those researches, we propose a consistent binormalized data-reusing least mean square (LMS) algorithm for identifying finite impulse response models whose input and output are corrupted by additive white noise. The proposed algorithm exploits the stochastic properties of the noisy input to compensate a bias of estimation which is occurred by additive white noise. Furthermore, the algorithm reuses input signal to overcome a decline of convergence performance with highly correlated input signal. The proposed algorithm has smaller steady-state estimation errors than the ordinary normalized LMS or BNDR-LMS and gets fast convergence rate compared to the consistent normalized LMS proposed in [7].

This paper is organized as follows. In Section II, we states the system model we concern. The statement of the problem and proposed algorithm is provided in Section III. The experimental results and concluding remarks are given in Section IV and Section V, respectively.

II. PRELIMINARY: SIGNAL MODEL

The adaptive filter is used to estimate the unknown system with FIR vector $\mathbf{h} \in \mathbb{R}^{M \times 1}$. The desired value $d(n)$ is represented as

$$d(n) = \mathbf{u}^T(n) \mathbf{h} + v_o(n) \quad (1)$$

where $\mathbf{u}(n) = [u(n), u(n-1), \dots, u(n-M+1)]^T$ denotes the noise-free input vector and $v_o(n)$ denotes observation noise with variance σ_o^2 . The noisy input vector of the adaptive filter is given by

$$\tilde{\mathbf{u}}(n) = \mathbf{u}(n) + \mathbf{v}_i(n) \quad (2)$$

where $\mathbf{v}_i(n) = [v_i(n), v_i(n-1), \dots, v_i(n-M+1)]^T$. The following assumptions are introduced.

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$$\begin{aligned}\frac{\lambda_1}{2} &= \frac{(d(n) - \mathbf{u}^T(n)\mathbf{w}(n))\mathbf{u}^T(n-1)\mathbf{u}(n-1) - (d(n-1) - \mathbf{u}^T(n-1)\mathbf{w}(n))\mathbf{u}^T(n)\mathbf{u}(n-1)}{\mathbf{u}^T(n)\mathbf{u}(n)\mathbf{u}^T(n-1)\mathbf{u}(n-1) - (\mathbf{u}^T(n)\mathbf{u}(n-1))^2 + \epsilon}, \\ \frac{\lambda_2}{2} &= \frac{(d(n-1) - \mathbf{u}^T(n-1)\mathbf{w}(n))\mathbf{u}^T(n)\mathbf{u}(n) - (d(n) - \mathbf{u}^T(n)\mathbf{w}(n))\mathbf{u}^T(n-1)\mathbf{u}(n)}{\mathbf{u}^T(n)\mathbf{u}(n)\mathbf{u}^T(n-1)\mathbf{u}(n-1) - (\mathbf{u}^T(n)\mathbf{u}(n-1))^2 + \epsilon}.\end{aligned}\quad (3)$$

A1) $v_i(n)$ and $v_o(n)$ are zero-mean white noise with variance σ_i^2 and σ_o^2 .

A2) $v_i(n)$ and $v_o(n)$ are independent and uncorrelated with $\mathbf{u}[n]$.

A3) The length M of the FIR and the input noise variance σ_i^2 is a priori known.

III. CONSISTENT BINORMALIZED DATA-REUSING LMS ALGORITHM

The binormalized data-reusing LMS algorithm obtains the projections of two consecutive gradient directions. The statement of the problem is that to minimize the squared Euclidean norm of the change $\delta\mathbf{w}(n+1) = \mathbf{w}(n+1) - \mathbf{w}(n)$ subjected to $\mathbf{u}^T(n)\mathbf{w}(n+1) = d(n)$ and $\mathbf{u}^T(n-1)\mathbf{w}(n+1) = d(n-1)$. The cost function for the problem is, then,

$$J(n) = \|\delta\mathbf{w}(n+1)\|^2 + \lambda_1 (d(n) - \mathbf{u}^T(n)\mathbf{w}(n+1)) + \lambda_2 (d(n-1) - \mathbf{u}^T(n-1)\mathbf{w}(n+1)). \quad (4)$$

The problem has unique solution as

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \frac{\lambda_1}{2}\mathbf{u}(n) + \frac{\lambda_2}{2}\mathbf{u}(n-1) \quad (5)$$

where λ_1 and λ_2 is given by (5)

However, we are only able to use a noisy measurement $\tilde{\mathbf{u}}(n)$ instead of $\mathbf{u}(n)$ when the input signal is noisy. Therefore, we exploit the following stochastic value with the assumptions (A1, A2):

$$\begin{aligned}E[\tilde{\mathbf{u}}(n)\tilde{\mathbf{u}}^T(n)] &= E[\mathbf{u}(n)\mathbf{u}^T(n)] + \sigma_i^2\mathbf{I} \\ E[\tilde{\mathbf{u}}^T(n)\tilde{\mathbf{u}}(n)] &= E[\mathbf{u}^T(n)\mathbf{u}(n)] + M\sigma_i^2 \\ E[\tilde{\mathbf{u}}(n+1)\tilde{\mathbf{u}}^T(n)] &= E[\mathbf{u}(n+1)\mathbf{u}^T(n)] \\ E[\tilde{\mathbf{u}}^T(n+1)\tilde{\mathbf{u}}(n)] &= E[\mathbf{u}^T(n+1)\mathbf{u}(n)]\end{aligned}\quad (6)$$

Further, approximating the expectation value with instantaneous values, then we have

$$\begin{aligned}\mathbf{u}(n)\mathbf{u}^T(n) &\approx \tilde{\mathbf{u}}(n)\tilde{\mathbf{u}}^T(n) - \sigma_i^2\mathbf{I} \\ \mathbf{u}^T(n)\mathbf{u}(n) &\approx \tilde{\mathbf{u}}^T(n)\tilde{\mathbf{u}}(n) - M\sigma_i^2 \\ \mathbf{u}(n+1)\mathbf{u}^T(n) &\approx \tilde{\mathbf{u}}(n+1)\tilde{\mathbf{u}}^T(n) \\ \mathbf{u}^T(n+1)\mathbf{u}(n) &\approx \tilde{\mathbf{u}}^T(n+1)\tilde{\mathbf{u}}(n)\end{aligned}\quad (7)$$

Substituting the λ_i of (5) and the approximations of (7) into the update equation (4) and introducing scaling factor

denoted by $\mu \in (0, 1]$, we obtain

TABLE I: PROPOSED CONSISTENT BINORMALIZED DATA-REUSING LMS ALGORITHM

Initialize: $\hat{\mathbf{w}}(1) = 0$,
For $n = 1, 2, \dots$,
$a = \tilde{\mathbf{u}}^T(n)\tilde{\mathbf{u}}(n)$ $b = \tilde{\mathbf{u}}^T(n-1)\tilde{\mathbf{u}}(n-1)$ $a = \tilde{\mathbf{u}}^T(n)\tilde{\mathbf{u}}(n-1)$ $d = \tilde{\mathbf{u}}^T(n)\mathbf{w}(n)$ $e = \tilde{\mathbf{u}}^T(n-1)\mathbf{w}(n)$ $f = (M-1)\sigma_i^2$ $e_1(n) = d(n) - \tilde{\mathbf{u}}(n)^T \hat{\mathbf{w}}(n)$ $e_2(n) = d(n-1) - \tilde{\mathbf{u}}(n-1)^T \hat{\mathbf{w}}(n)$ $A = e_1(b-f) - e_2c - \sigma_i^2d$ $B = e_2(a-f) - e_1c - \sigma_i^2e$ $C = \sigma_i^2(a+b-2f)$ $D = ab - c^2 - M\sigma_i^4 + \epsilon$ $\hat{\mathbf{w}}(n+1) = \hat{\mathbf{w}}(n) + \mu(A\tilde{\mathbf{u}}(n) + B\tilde{\mathbf{u}}(n-1) + C\hat{\mathbf{w}}(n)) / D$
The step-size μ is pre-defined. M denotes the length of the unknown FIR and ϵ is small value

$$\hat{\mathbf{w}}(n+1) = \hat{\mathbf{w}}(n) + \mu(A\mathbf{u}(n) + B\mathbf{u}(n-1) + C\hat{\mathbf{w}}(n)) / D \quad (8)$$

where

$$\begin{aligned}A &= e_1(n)(\tilde{\mathbf{u}}^T(n-1)\tilde{\mathbf{u}}(n-1) - (M-1)\sigma_i^2) \\ &\quad - e_2(n)\tilde{\mathbf{u}}^T(n)\tilde{\mathbf{u}}(n-1) - \sigma_i^2\tilde{\mathbf{u}}^T(n)\hat{\mathbf{w}}(n), \\ B &= e_2(n)(\tilde{\mathbf{u}}^T(n)\tilde{\mathbf{u}}(n) - (M-1)\sigma_i^2) \\ &\quad - e_1(n)\tilde{\mathbf{u}}^T(n)\tilde{\mathbf{u}}(n-1) - \sigma_i^2\tilde{\mathbf{u}}^T(n-1)\hat{\mathbf{w}}(n), \\ C &= \sigma_i^2(\tilde{\mathbf{u}}^T(n-1)\tilde{\mathbf{u}}(n-1) + \tilde{\mathbf{u}}^T(n)\tilde{\mathbf{u}}(n) - 2(M-1)\sigma_i^2), \\ D &= \tilde{\mathbf{u}}^T(n)\tilde{\mathbf{u}}(n)\tilde{\mathbf{u}}^T(n-1)\tilde{\mathbf{u}}(n-1) - (\tilde{\mathbf{u}}^T(n)\tilde{\mathbf{u}}(n-1))^2 \\ &\quad - M\sigma_i^4 + \epsilon,\end{aligned}\quad (9)$$

And
$$\begin{aligned}e_1(n) &= d(n) - \tilde{\mathbf{u}}(n)^T \hat{\mathbf{w}}(n), \\ e_2(n) &= d(n-1) - \tilde{\mathbf{u}}(n-1)^T \hat{\mathbf{w}}(n).\end{aligned}$$

The proposed algorithm is summarized in Table I

IV. EXPERIMENTAL RESULTS

In order to compare the performance of the proposed algorithm with other adaptive filtering algorithms, we carried the following FIR identification experiment. The FIR of the unknown system is generated by randomly with the length of

16. The input signal $u(n)$ is generated as a white, zero-mean, Gaussian random sequence or obtained by filtering it through a first-order system $G_1(z) = 1/(1 - 0.8z^{-1})$.

The signal to noise ratio (SNR) of the input is set to 20dB which is defined by and the SNR of the measurement is set to 30dB which is defined by

$$\text{SNR}_i = 10 \log_{10} \sum_n u(n)^2 / \sum_n v_i(n)^2, \quad (10)$$

$$\text{SNR}_o = 10 \log_{10} \sum_n (\tilde{\mathbf{u}}^T(n) \mathbf{h})^2 / \sum_n v_o(n)^2 \quad (11)$$

The estimation error as the performance measure is obtained by using the mean square deviation (MSD) defined as $\text{MSD}(n) = (\mathbf{h} - \hat{\mathbf{w}}(n))^T (\mathbf{h} - \hat{\mathbf{w}}(n)) / \mathbf{h}^T \mathbf{h}$ and the simulation results are obtained by ensemble averaging over 100 independent trials.

We tested both a white input sequence and correlated input sequence and the compared performance of the proposed algorithm is illustrated in Fig. 1 and Fig 2, respectively. We compared the proposed algorithm with several related studies of the NLMS, the consistent NLMS (C-NLMS), and BNDR-LMS [7], [8]. As expected the performance of the proposed algorithm is very similar with that of the C-NLMS when we use a white input sequence and they are better than the performance of NLMS and BNDR-LMS Fig 1.

Fig. 1. Mean-square deviation of NLMS, C-NLMS, BNDR-LMS, and the proposed algorithm using white input sequences

Fig. 2. Mean-square deviation of NLMS, C-NLMS, BNDR-LMS, and the proposed algorithm using correlated input sequences

However, when the colored input which is generated by first order AR model was used, the proposed algorithm achieved fast convergence speed compared to C-NLMS while having smaller steady-state estimation errors than that of NLMS and BNDR_LMS (Fig 2).

V. CONCLUSION

A consistent binormalized data-reusing least mean square (LMS) algorithm for identifying FIR models whose input and output are corrupted by additive white noise was proposed. By exploiting the stochastic properties of the noisy input, the proposed algorithm compensated a bias of estimation. Moreover, in order to improve convergence performance, the input data was reused. The simulation results of an adaptive FIR estimation showed that the proposed algorithm achieved consistent estimation with noisy input signal. Furthermore, the proposed algorithm had faster convergence rate than the ordinary consistent LMS algorithms while having smaller steady-state estimation errors than that of NLMS and BNDR_LMS when the input signal was highly correlated.

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