A Novel Particle Swarm Optimization with Passive Congregation via Chaotic Sequences

Reza Gholipour, Hamed Mojallali, and Seyed Mohammad Kazem Akhlaghi

Abstract—This paper proposes a novel particle swarm optimization with passive congregation (PSOPC) algorithm based on the chaos phenomenon. The proposed Chaotic PSOPC (CPSOPC) algorithm utilizes chaotic sequences to improve the global searching by escaping the local optima. This algorithm overcomes the disadvantages of premature convergence in the PSOPC. We compared the proposed algorithm with PSOPC for eight benchmark functions with 10 and 20 dimensions. Simulation results indicate that the CPSOPC improves the global convergence of PSOPC. Furthermore, the proposed algorithm improves the accuracy and efficiency of searches compared with PSOPC and increases the convergence speed of PSOPC.

Index Terms—Chaotic sequences, particle swarm optimization, passive congregation, logistic map.

I. INTRODUCTION

Swarm Intelligence (SI) methods are based around the study of collective behavior in decentralized, self-organized systems. SI systems are typically made up of a population of simple agents interacting locally with one another and with their environment. Although there is no centralized control structure dictating how individual agents should behave, local interactions between such agents often lead to the emergence of a global behavior.

Two of the most successful SI techniques modeled on the behavior of natural systems are ant colony optimization (ACO) proposed by Dorigo and Gambardella [1] and particle swarm optimization (PSO) proposed by Kennedy and Eberhart [2].

In ACO artificial ants build solutions by traversing a problem space. Similar to real ants, they deposit artificial pheromone on the workspace in a manner that makes it possible for future ants to build better solutions. In real ant colonies the pheromone is used to find the shortest path to food. Using ACO, finite size colonies of artificial ants communicate with each other via artificial pheromones to find quality solutions to optimizations problems. ACO has been applied to a wide range of optimization problems such as the traveling salesman problem, and routing and load balancing in packet switched network [1].

The PSO approach utilizes a population based stochastic

optimization algorithm [2]. It was inspired from the computer simulation of the social behavior of bird flocking by Reynolds [3]. In PSO, a set of randomly generated agents propagate in the design space towards the optimal solution over a number of iterations. Each agent has a memory of its best position and the swarm's best solution. PSO is also easy to implement and the fact that no gradient information is required makes it a good candidate for a wide variety of optimization problems [4]. Recently, PSO has been successfully used in the optimization of many engineering fields. ZaferBingul ,OguzhanKarahan, introduced the PSO based tuning method for Fuzzy Logic Controller(FLC) to control the 2 DOF robot trajectory[5].Acharjee and Goswami developed a robust, nondivergent power flow method by using PSO technique[6]. HassenT.Dorrah et al. proposed PSO-BELBIC for two-coupled distillation column process. The PSO_BELBIC improves the time domain parameters of all loops of the process[7]. PSO and its variants widely are applied for tuning of PID controller parameters [8]-[12].

In standard PSO and its variants like: global version of SPSO (GPSO), local version of SPSO (LPSO) and PSO with a constriction factor (CPSO), exchange of information between the particles is low and particles will capture in the local optimum. In PSOPC, by introducing of passive congregation, the exchange of information between particles arises insignificantly. It leads to an improvement in PSO performance [13].

In standard PSO and PSOPC, all particles have the tendency to follow the current best position that may be a local optimum or a position near the local optimum, so that all particles will concentrate to a small region and cannot search in global solution space again. Thus, particles will capture in the local optimum and the premature convergence phenomena occur.

In order to, improve the PSOPC performance, a Chaotic PSOPC is delivered in this paper. In this method, adding chaotic sequences with ergodic, irregular, dynamic and stochastic properties in PSOPC the global convergence is improved.

The rest of the paper is organized as follows: Section 2 describes typical PSO. In section 3 PSOPC is described. Section 4 presents the proposed CPSOPC algorithm. Section 5 expresses the result from CPSOPC while compared with other algorithms for benchmark functions. Finally, the last section gives an abbreviated conclusion about this study.

II. PARTICLE SWARM OPTIMIZATION

The PSO is an evolutionary computational model, a stochastic search technique based on swarm intelligence. Social behavioral pattern of organisms such as bird flocking

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and fish schooling inspired them to look into the effect of collaboration of species when obtaining their goals as a group. Dynamics of bird flocking resulted in the possibilities of utilizing this behavior as an optimization tool. These have been used to solve a range of optimization problems.

Suppose that a swarm contains m particles moving around in a d -dimensional search space. The i th particle at the t th iteration has a position $X_i(t) = (X_{i1}(t), X_{i2}(t), \dots, X_{id}(t))$, which is a feasible solution of an optimal problem, a velocity $V_i(t) = (V_{i1}(t), V_{i2}(t), \dots, V_{id}(t))$, and the best solution ever achieved so far by itself, individual best position P_{best} , which is represented by $P_i(t) = (P_{i1}(t), P_{i2}(t), \dots, P_{id}(t))$. The best solution ever achieved so far by the whole swarm, global best position G_{best} , which is represented by $P_g(t) = (P_{g1}(t), P_{g2}(t), \dots, P_{gd}(t))$. Then, the velocity and the position of the i th particle in the k th dimension at the next iteration will be calculated as:

$$\begin{aligned} V_{ik}(t+1) &= w * V_{ik}(t) + c_1 * r_1 * (P_{ik}(t) - X_{ik}(t)) \\ &+ c_2 * r_2 * (P_{gk}(t) - X_{ik}(t)) \\ X_{ik}(t+1) &= X_{ik}(t) + V_{ik}(t+1) \end{aligned} \quad (1)$$

where w is inertia weight, c_1 and c_2 are two positive constants, called cognitive learning rate and social learning rate, respectively, and r_1 and r_2 are two uniformly random numbers from interval $[0,1]$.

A linearly decreasing inertia weight [14], [15] was first introduced by Shi and Eberhart in 1998. At the beginning of the searching process, a larger inertia factor is applied for global exploration. During the search, inertia weight is becoming smaller and smaller for local exploitation, which leads to significant improvement in the performance of classical PSO. The linearly decreasing inertia weight is modified as:

$$w(t) = w_{\max} - \frac{w_{\max} - w_{\min}}{t_{\max}} * t \quad (3)$$

And the velocity is modified as:

$$\begin{aligned} V_{ik}(t+1) &= w(t) * V_{ik}(t) + c_1 * r_1 * (P_{ik}(t) - X_{ik}(t)) \\ &+ c_2 * r_2 * (P_{gk}(t) - X_{ik}(t)) \end{aligned} \quad (4)$$

where $w_{\max}$ is the initial inertia weight, $w_{\min}$ is the final inertia weight, and $t_{\max}$ is the number of maximum iteration.

Normally, $w_{\max}$ is set to 0.9 and $w_{\min}$ is set to 0.4.

III. PARTICLE SWARM OPTIMIZATION WITH PASSIVE CONGREGATION

The PSO can be enhanced by one type of social behaviors such as bird flocking, fish schooling and insects swarming, which are considered as congregation. This behavior is connected with grouping by social forces which is the source of attraction. The congregation includes active congregation

and passive congregation. The latter is an attraction of an individual to other group members but not a display of social behavior [13]. Fish schooling is one of the representative types of passive congregation and the PSO is inspired by it. Adding the passive congregation model to the SPSPSO may increase its performance. He et al. proposed a hybrid PSO with passive congregation (PSOPC) as follows [13]:

$$\begin{aligned} V_{iQ}(t+1) &= w V_{iQ}(t) + c_1 r_1 (P_{iQ}(t) - X_{iQ}(t)) + \\ &c_2 r_2 (P_{gQ}(t) - X_{iQ}(t)) + c_3 r_3 (R_{iQ}(t) - X_{iQ}(t)) \\ X_{iQ}(t+1) &= X_{iQ}(t) + V_{iQ}(t+1) \end{aligned} \quad (5)$$

where R_i is a particle selected randomly from the swarm, c_3 is the passive congregation coefficient and r_3 is a uniform random sequences in the range $(0,1)$: $r_3 \sim U[0,1]$.

IV. CHAOTIC PARTICLE SWARM OPTIMIZATION WITH PASSIVE CONGREGATION

In the field of engineering, it is well recognized that chaos theory can be applied as a very useful technique in practical application. The chaotic system can be described by a phenomenon, in which a small change in the initial condition will lead to nonlinear change in future behavior, besides that the system exhibits distinct behaviors under different phases, i.e. stable fixed points, periodic oscillations, bifurcations, and ergodicity. Chaos is also a common nonlinear phenomenon with much complexity and is similar to randomness. Chaos is typically highly sensitive to the initial values and thus provides great diversity based on the ergodic property of the chaos phase, which transits every state without repetition in certain ranges. It is generated through a deterministic iteration formula. Due to these characteristics, chaos theory can be applied in optimization.

The Chaotic particle swarm optimization with passive congregation (CPSOPC) algorithm is based on PSOPC and chaotic sequences. In the PSOPC convergence, the parameters w, c_1, c_2, r_1, r_2 and r_3 are mainly the effective key factor [13, 16 and 17]. In CPSOPC, sequences generated by the logistic map [18] substitute the random parameters r_1 and r_2 in PSOPC. The parameters r_1 and r_2 are modified by the logistic map based on the following equation.

$$Cr(t+1) = 4Cr(t)(1 - Cr(t)), Cr(t) \in [0,1] \quad (7)$$

In Eq.7, $Cr(0)$ is generated randomly for each independent run, with $Cr(0)$ not being equal to $\{0, 0.25, 0.5, 0.75, 1\}$. Chaotic sequences Cr are sensitive to the initial conditions of $Cr(0)$. The chaotic values of Cr for $Cr(0) = 0.0001$ and $Cr(0) = 0.00011$ are shown in Fig. 1.

Dependence of chaotic sequences Cr to initial conditions is depicted in Figure 1. As it shows, with little change in initial condition of $Cr(0)$, chaotic sequences Cr will lead to huge changes.

The velocity and position update equations for CPSOPC, can be formulated as:

$$V_{iQ}(t+1) = wV_{iQ}(t) + c_1Cr_1(P_{iQ}(t) - X_{iQ}(t)) + c_2Cr_2(P_{gQ}(t) - X_{iQ}(t)) + c_3r_3(R_{iQ}(t) - X_{iQ}(t)) \quad (8)$$

$$X_{iQ}(t+1) = X_{iQ}(t) + V_{iQ}(t+1) \quad (9)$$

where $Cr_1, Cr_2 \in [0,1]$ are functions based on results of logistic map. The flowchart of proposed algorithm CPSOPC is described as in Fig. 2.

V. NUMERICAL SIMULATIONS

A. Benchmark Functions

In order to test the performance of proposed algorithm on numerical function optimization, eight numerical benchmark functions are used here.

Fig. 1. Dynamics of logistic map

The first function is the Sphere function described by

$$f_1(\vec{x}) = \sum_{i=1}^D x_i^2 \quad (10)$$

where the initial range of $\vec{x}$ is $[-100,100]^D$. The minimum solution of the Sphere function is $\vec{x}^* = [0,0,...,0]$ and $f_1(\vec{x}^*) = 0$.

Fig. 2. Flowchart of CPSOPC algorithm

The second function is the Griewank function described by

$$f_2(\vec{x}) = \frac{1}{4000} \left(\sum_{i=1}^D (x_i - 100)^2 \right) - \left(\prod_{i=1}^D \cos \left(\frac{x_i - 100}{\sqrt{i}} \right) \right) + 1 \quad (11)$$

where the initial range of $\vec{x}$ is $[-600,600]^D$. The minimum solution of the Griewank function is $\vec{x}^* = [100,100,...,100]$ and $f_2(\vec{x}^*) = 0$.

The third function is the Ackley function described by

$$f_3(\vec{x}) = 20 + e - 20 \exp\left(-0.2\sqrt{\frac{1}{D} \sum_{i=1}^D x_i^2}\right) - \exp\left(\frac{1}{D} \sum_{i=1}^D \cos(2\pi x_i)\right) \quad (12)$$

where the initial range of $\vec{x}$ is $[-32.768, 32.768]^D$. The minimum solution of the Rastrigin function is $\vec{x}^* = [0, 0, \dots, 0]$ and $f_3(\vec{x}^*) = 0$.

The fourth function is the Rosenbrock function described by

$$f_4(\vec{x}) = \sum_{i=1}^{D-1} 100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2 \quad (13)$$

where the initial range of $\vec{x}$ is $[-50, 50]^D$. The minimum solution of the Rosenbrock function is $\vec{x}^* = [1, 1, \dots, 1]$ and $f_4(\vec{x}^*) = 0$.

The fifth function is the Rastrigin function described by

$$f_5(\vec{x}) = \sum_{i=1}^D (x_i^2 - 10 \cos(2\pi x_i) + 10) \quad (14)$$

where the initial range of $\vec{x}$ is $[-5.12, 5.12]^D$. The minimum solution of the Rastrigin function is $\vec{x}^* = [0, 0, \dots, 0]$ and $f_5(\vec{x}^*) = 0$.

The sixth function is the generalized Schaffer function described by

$$f_6(\vec{x}) = 0.5 + \frac{\sin^2\left(\sqrt{\frac{1}{D} \sum_{i=1}^D x_i^2}\right) - 0.5}{\left(1 + 0.001 \left(\sqrt{\frac{1}{D} \sum_{i=1}^D x_i^2}\right)\right)^2} \quad (15)$$

where $\vec{x} = [x_1, x_2, \dots, x_D]$, the initial range of $\vec{x}$ is $[-100, 100]^D$ and D denotes the dimension of the solution space. The minimum solution of the Schaffer function is $\vec{x}^* = [0, 0, \dots, 0]$ and $f_6(\vec{x}^*) = 0$.

The seventh function is the Schwefel function described by

$$f_7 = 418.9829D - \sum_{i=1}^D x_i \sin\left(\sqrt{|x_i|}\right) \quad (16)$$

$$f_8 = \frac{\pi}{30} \left\{ 10 \sin^2(\pi y_1) + \sum_{i=1}^{29} (y_i - 1)^2 \times \left[1 + 10 \sin^2(\pi y_{i+1}) \right] + (y_n - 1)^2 \right\} + \sum_{i=1}^{30} u(x_i, 10, 100, 4) \quad (17)$$

where $\vec{x} = [x_1, x_2, \dots, x_D]$, the initial range of $\vec{x}$ is $[-500, 500]^D$ and D denotes the dimension of the solution space. The minimum solution of the Schaffer function is $\vec{x}^* = [420.9687, 420.9687, \dots, 420.9687]$ and $f_7(\vec{x}^*) = 0$.

The eighth function is the Penalized function described by where

$$u(x_i, a, k, m) = \begin{cases} k(x_i - a)^m & x_i > a \\ 0 & -a \leq x_i \leq a \\ k(-x_i - a)^m & x_i < -a \end{cases} \quad (18)$$

And

$$y_i = 1 + \frac{1}{4}(x_i + 1) \quad (19)$$

where $\vec{x} = [x_1, x_2, \dots, x_D]$, the initial range of $\vec{x}$ is $[-50, 50]^D$ and D denotes the dimension of the solution space. The minimum solution of the Schaffer function is $\vec{x}^* = [-1, -1, \dots, -1]$ and $f_8(\vec{x}^*) = 0$.

B. Experimental Setting

The population size of three algorithms was set at 70. The acceleration constants C_1 and C_2 in CPSOPC and PSO were set 2 and in PSOPC was set 0.5. In both CPSOPC and PSOPC the passive congregation coefficient C_3 linearly increases from 0.4 to 0.6. The inertia weight w for three algorithms started from 0.9 and decreased linearly to 0.7. The number of iteration allowed in each run was set to 500 for dimension 10 and 1000 for dimension 20. All experiments were repeated for 80 runs. In CPSOPC algorithm the number of chaos iteration K_{chaos} was set to be 15.

TABLE I: COMPARISON OF ACQUIRED RESULTS FROM CPSOPC WITH PSOPC AND PSO

Test function	Dim	CPSOPC		PSOPC		PSO	
		Mean	SD	Mean	SD	Mean	SD
f_1	10	5.4397e-020	8.6844e-020	1.4678e-005	4.3686e-005	0.0364	0.0890
	20	1.0506e-011	1.9929e-011	168.5687	101.7761	501.0667	2.2361e+003
f_2	10	0.0888	0.0380	0.1350	0.0913	0.2752	0.1804
	20	0.0255	0.0172	3.5740	2.9779	4.6275	14.0996
f_3	10	8.4337e-010	2.1477e-009	0.8005	0.9435	1.3316	0.9721
	20	2.2275e-004	2.3721e-004	1.4760	0.8945	5.2445	6.5278
f_4	10	0.0011	0.0041	45.1296	116.2293	60.3922	118.4356
	20	10.7270	1.2380	202.7378	575.9683	246.8969	593.4850
f_5	10	3.4326	2.3384	8.7629	4.8263	12.7924	8.0170
	20	16.9646	12.8535	47.7107	14.9805	51.1067	15.9686
f_6	10	0.0097	3.8005e-010	0.0269	0.0183	0.0502	0.0219
	20	0.0097	1.2488e-010	0.1525	0.0475	0.3632	0.0616
f_7	10	922.9784	192.6075	970.3547	333.5117	1.0028e+003	276.7715
	20	2.5195e+003	438.1564	2.7191e+003	505.9755	2.8186e+003	1.0301e+003
f_8	10	3.0861e-020	1.2912e-019	0.0456	0.1046	0.0856	0.2419
	20	9.0095e-011	2.4246e-010	0.6139	0.6274	1.4461	0.8485

C. Experimental Results

The experimental results consist of the mean and the standard deviations of the function values found in 80 runs are listed in Table I. From the results of Table1, CPSOPC algorithm outperforms the PSOPC algorithm for all the benchmark functions. Mean and std illustrate the CPSOPC has extraordinary convergence and robustness.

Various curves, in Fig. 2-9 illustrate the mean fitness values resulted by the three algorithms with respect to iterations.

Fig. 2. Convergence curves of CPSOPC, PSOPC and PSO algorithms for the Sphere function for D=10 and D=20

Fig. 3. Convergence curves of CPSOPC, PSOPC and PSO algorithms for the Griewank function for D=10 and D=20

According to Fig. 2-9, the CPSOPC can find the best fitness with fast convergence speed. In fact with adding chaotic sequences to PSOPC, the search behavior is improved and the entrapment of particles in a local optimal is prevented.

Fig. 4. Convergence curves of CPSOPC, PSOPC and PSO algorithms for the Ackley function for D=10 and D=20

Fig. 5. Convergence curves of CPSOPC, PSOPC and PSO algorithms for the Rosenbrock function for D=10 and D=20

Fig. 6. Convergence curves of CPSOPC, PSOPC and PSO algorithms for the Rastrigin function for D=10 and D=20

Fig. 8. Convergence curves of CPSOPC, PSOPC and PSO algorithms for the Schwefel function for D=10 and D=20

Fig. 7. Convergence curves of CPSOPC, PSOPC and PSO algorithms for the Schaffer function for D=10 and D=20

Fig. 9. Convergence curves of CPSOPC, PSOPC and PSO algorithms for the Penalized function for D=10 and D=20

D. Discussion

In CPSOPC, a chaotic map was embedded to determine the PSOPC parameters r_1 and r_2 . The PSO and PSOPC parameters r_1 and r_2 cannot ensure optimal ergodicity in the search space because they are absolutely random. i.e. the r_1 and r_2 are generated by a linear congruential generator (LCG) with a random seed. The generated sequence of LCG consists of pseudo-random numbers that have periodic characteristics [19]. Furthermore, the generated sequence of a logistic map also consists of pseudo-random numbers, but there are no fixed points, periodic orbits, or quasi periodic orbits in the behavior of the chaos system [20].

As a result, the system can avoid being entrapment in local optima. In terms of mean fitness values, figures 2–9 indicate that CPSOPC outperformed PSOPC on the Sphere, Griewank, Ackley, Rosenbrock, Rastrigin, Schaffer, Schwefel and Penalized functions.

VI. CONCLUSION

In this paper, a chaotic particle swarm optimization with passive congregation (CPSOPC), based on the particle swarm optimization with passive congregation (PSOPC), and chaotic sequences is presented to improve the premature convergence in the PSOPC method. The Comparison between CPSOPC and PSOPC has shown that the proposed algorithm improves the global search of PSOPC by escaping the local optima. The CPSOPC algorithm tested on eight benchmark functions and results show the efficiency of the proposed algorithm.

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