Multi Objective Dispatch Problem with Valve Point Effect Loading of Fuel Cost and Emission Criterion

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Abstract—In most of the publications the dispatch problem is formulated using fuel cost and emission function without considering valve point effect. The paper formulates a multi objective dispatch problem with valve point effect considered in both fuel cost and emission function. Most of the literature uses penalty factors such as Max-Max, Min-Min, Average, and common to combine both fuel cost and emission function in one criterion. In addition to this, Max-Min and, Min-Max penalty factors are proposed in the paper. Lagrange's method is formulated to solve the combined economic emission dispatch problem with proposed penalty factors. The considered dispatch problem is solved for the IEEE test system of five generators by using Matlab software. The results show that Min-Max penalty factor method gives minimum fuel cost and emission values in comparison to other penalty factors.

Index Terms—Power system optimization, economic dispatch, multi objective dispatch problem, penalty factors, valve point effect loading, and lagrange's method.

I. INTRODUCTION

Optimization of the power system plays a major role in thermal power plants energy production. The challenges of the engineers are to optimize the real power of the generator and to minimize the fuel cost and emission of the power plant. Most of the literature papers concentrate on combined economic emission dispatch problem without considering valve point effect loading. Actually the power plant has multiple steam valves, so it is necessary to consider the valve point effect loading. Therefore solution of the dispatch problem is also more accurate when the dispatch problem with valve point effect loading is solved. The methods used or developed to solve the multi objective dispatch problems formulated by using penalty factors are described in the paper. The combined fuel cost function and emission function without valve point effect is formulated by Weighted Sum Method (WSM) using PSO algorithm in [17], [18] and Evolutionary programming in [19]. The combined fuel cost function and emission function without valve point effect is formulated by various price penalty factors approaches using recursive algorithm in [20]. Max-Max price penalty factor method using quadratic programming in [21], security constrained unit commitment

problem algorithm in [22], sequential approach with matrix frame work in [23], PSO in [24], Improved back propagation neural network in [25], and Lagrange's method in [26]. The combined fuel cost function without valve point effect and emission function with valve point effect is formulated using PSO algorithm in [15]. The combined fuel cost function with valve point effect and emission function with valve point effect is formulated using Improved PSO algorithm with carbon tax considered in [14], Improved pattern search in [16] using WSM method, and Lagrange's method in [27]. Most of the literature papers reported that the multi objective dispatch problem is solved using the WSM and Max-Max penalty factors. The paper proposes two penalty factors Min-Max and Max-Min in order to solve the multi-criteria dispatch problem with or without valve point effect as described by equation (5) and (14) respectively. The optimization problem is solved on the basis of the Lagrange's method. The paper consists of five parts. Part 1 describes the introduction of the dispatch problem, economic dispatch problem formulation is presented in part 2, part 3 covers Lagrange's method for multi-criteria dispatch problem, simulation results and conclusion are given in part 4 and part 5 respectively.

II. ECONOMIC DISPATCH PROBLEM FORMULATION

A. Fuel Cost Function

The fuel cost of the economic dispatch problem can be formulated as

$$F_C = \sum_{i=1}^n F_i(P_i) = \sum_{i=1}^n (a_i P_i^2 + b_i P_i + c_i) \quad [\$ / hr] \quad (1)$$

where

- F_C - Total Fuel Cost
- $F_i(P_i)$ - Fuel cost of the i^{th} generator
- P_i - Real power generation of unit i
- a_i, b_i, c_i - Cost coefficients of generating for unit i
- n - Number of generating units

Thermal power plants have multiple steam valves. To accurately evaluate the fuel cost function, the valve point effect loading is considered as follows:

$$F_{CVP} = \sum_{i=1}^n F_i(P_i) = \sum_{i=1}^n \left[(a_i P_i^2 + b_i P_i + c_i) + |e_i \sin(f_i(P_{i,\min} - P_i))| \right] [\$ / hr] \quad (2)$$

where

- F_{CVP} - Total Fuel Cost with valve point effect
- e_i, f_i - Valve point effect cost coefficients of generating

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unit i

B. Emission Function

The emission of the thermal power plant can be formulated as a second order polynomial function as

$$E_T = \sum_{i=1}^n (\alpha_i P_i^2 + \beta_i P_i + \gamma_i) \text{ [Kg/hr]} \quad (3)$$

where

E_T - Total emission

$\alpha_i, \beta_i, \gamma_i$ - Emission coefficients of generating unit i

The emission equation (3) can be formulated with inserting the valve point effect as follows:

$$E_{TVP} = \sum_{i=1}^n (\alpha_i P_i^2 + \beta_i P_i + \gamma_i) + |\delta_i \sin(\eta_i (P_{i,\min} - P_i))| \text{ [Kg/hr]} \quad (4)$$

where

E_{TVP} Total emission with valve point effect

δ_i, η_i Valve point effect emission coefficients of generating unit i

C. Multi-Criteria Objective Function without Valve Point Effect Loading Using Penalty Factors

The combined economic emission dispatch problem is formulated as follows:

$$F_T = \sum_{i=1}^n [(a_i P_i^2 + b_i P_i + c_i) + h_i (\alpha_i P_i^2 + \beta_i P_i + \gamma_i)] \text{ [$/hr]} \quad (5)$$

where

h_i - Price penalty factor

D. Formulation of Price Penalty Factors of Multi-Criteria Dispatch Problem without Valve Point Effect

The fuel cost and emission function are correlated with any one of the following price penalty factors. The paper uses six types of penalty factor for the multi-criteria dispatch problem, as follows:

The ratio of minimum fuel cost and maximum emission is called Min-Max Penalty factor [26], [27]. It can be formulated as

$$h_i = \frac{(a_i P_{i,\min}^2 + b_i P_{i,\min} + c_i)}{(\alpha_i P_{i,\max}^2 + \beta_i P_{i,\max} + \gamma_i)} \text{ [$/kg]} \quad (6)$$

The ratio of maximum fuel cost and maximum emission is called Max- Max Penalty factor [20] to [25]. It can be formulated as

$$h_i = \frac{(a_i P_{i,\max}^2 + b_i P_{i,\max} + c_i)}{(\alpha_i P_{i,\max}^2 + \beta_i P_{i,\max} + \gamma_i)} \text{ [$/kg]} \quad (7)$$

The ratio of minimum fuel cost and minimum emission is called Min- Min Penalty factor [20]. It can be formulated as

$$h_i = \frac{(a_i P_{i,\min}^2 + b_i P_{i,\min} + c_i)}{(\alpha_i P_{i,\min}^2 + \beta_i P_{i,\min} + \gamma_i)} \text{ [$/kg]} \quad (8)$$

The ratio of maximum fuel cost and minimum emission is

called Max- Min Penalty factor [26], [27]. It can be formulated as

$$h_i = \frac{(a_i P_{i,\max}^2 + b_i P_{i,\max} + c_i)}{(\alpha_i P_{i,\min}^2 + \beta_i P_{i,\min} + \gamma_i)} \text{ [$/kg]} \quad (9)$$

The average of the Min-Max, Max-Max, Min-Min, and Max-Min is called the average penalty factor [20]. It can be formulated as

$$h_i = \frac{\left[\frac{(a_i P_{i,\min}^2 + b_i P_{i,\min} + c_i)}{(\alpha_i P_{i,\max}^2 + \beta_i P_{i,\max} + \gamma_i)} + \frac{(a_i P_{i,\max}^2 + b_i P_{i,\max} + c_i)}{(\alpha_i P_{i,\max}^2 + \beta_i P_{i,\max} + \gamma_i)} + \frac{(a_i P_{i,\min}^2 + b_i P_{i,\min} + c_i)}{(\alpha_i P_{i,\min}^2 + \beta_i P_{i,\min} + \gamma_i)} + \frac{(a_i P_{i,\max}^2 + b_i P_{i,\max} + c_i)}{(\alpha_i P_{i,\min}^2 + \beta_i P_{i,\min} + \gamma_i)} \right]}{4} \text{ [$/kg]} \quad (10)$$

Apart from the penalty factors discussed above, a common penalty factor [20] exists for the multi-criteria dispatch problem. It can be formulated as follows

$$h_i = \frac{\sum_{i=1}^n \left[\frac{(a_i P_{i,\min}^2 + b_i P_{i,\min} + c_i)}{(\alpha_i P_{i,\max}^2 + \beta_i P_{i,\max} + \gamma_i)} + \frac{(a_i P_{i,\max}^2 + b_i P_{i,\max} + c_i)}{(\alpha_i P_{i,\max}^2 + \beta_i P_{i,\max} + \gamma_i)} + \frac{(a_i P_{i,\min}^2 + b_i P_{i,\min} + c_i)}{(\alpha_i P_{i,\min}^2 + \beta_i P_{i,\min} + \gamma_i)} + \frac{(a_i P_{i,\max}^2 + b_i P_{i,\max} + c_i)}{(\alpha_i P_{i,\min}^2 + \beta_i P_{i,\min} + \gamma_i)} \right]}{4n} \text{ [$/kg]} \quad (11)$$

E. Multi-Criteria Objective Function with Valve Point Effect Loading Using Penalty Factors

A couple of cases can be considered:

Fuel cost function with valve point effect loading and emission function without valve point effect:

$$F_T = \sum_{i=1}^n \left[((a_i P_i^2 + b_i P_i + c_i) + |e_i \sin(f_i (P_{i,\min} - P_i))|) + h_i (\alpha_i P_i^2 + \beta_i P_i + \gamma_i) \right] \text{ [$/hr]} \quad (12)$$

Fuel cost function without valve point effect loading and emission function with valve point effect:

$$F_T = \sum_{i=1}^n \left[(a_i P_i^2 + b_i P_i + c_i) + h_i \left((\alpha_i P_i^2 + \beta_i P_i + \gamma_i) + |\delta_i \sin(\eta_i (P_{i,\min} - P_i))| \right) \right] \text{ [$/hr]} \quad (13)$$

Fuel cost function and emission function with valve point effects:

$$F_{TVP} = \sum_{i=1}^n \left[((a_i P_i^2 + b_i P_i + c_i) + |e_i \sin(f_i (P_{i,\min} - P_i))|) + h_i \left((\alpha_i P_i^2 + \beta_i P_i + \gamma_i) + |\delta_i \sin(\eta_i (P_{i,\min} - P_i))| \right) \right] \text{ [$/hr]} \quad (14)$$

F. Formulation of Price Penalty Factors of Multi-Criteria Dispatch Problem with Valve Point Effect

The fuel cost and emission functions with valve point effect are correlated with any one of the following price penalty factors. The paper uses six types of penalty factors

for the multi-criteria dispatch problem as follows:

The ratio of minimum fuel cost and maximum emission with valve point effect is called Min-Max Penalty factor. It can be formulated as

$$h_i = \frac{(a_i P_{i,\min}^2 + b_i P_{i,\min} + c_i) + |e_i \sin(f_i (P_{i,\min} - P_{i,\min}))|}{(\alpha_i P_{i,\max}^2 + \beta_i P_{i,\max} + \gamma_i) + |\delta_i \sin(\eta_i (P_{i,\max} - P_{i,\max}))|} \quad [\$ / kg] \quad (15)$$

The ratio of maximum fuel cost and maximum emission with valve point effect is called Max- Max Penalty factor. It can be formulated as

$$h_i = \frac{(a_i P_{i,\max}^2 + b_i P_{i,\max} + c_i) + |e_i \sin(f_i (P_{i,\max} - P_{i,\max}))|}{(\alpha_i P_{i,\max}^2 + \beta_i P_{i,\max} + \gamma_i) + |\delta_i \sin(\eta_i (P_{i,\max} - P_{i,\max}))|} \quad [\$ / kg] \quad (16)$$

The ratio of minimum fuel cost and minimum emission with valve point effect is called Min- Min Penalty factor. It can be formulated as

$$h_i = \frac{(a_i P_{i,\min}^2 + b_i P_{i,\min} + c_i) + |e_i \sin(f_i (P_{i,\min} - P_{i,\min}))|}{(\alpha_i P_{i,\min}^2 + \beta_i P_{i,\min} + \gamma_i) + |\delta_i \sin(\eta_i (P_{i,\min} - P_{i,\min}))|} \quad [\$ / kg] \quad (17)$$

The ratio of maximum fuel cost and minimum emission with valve point effect is called Max- Min Penalty factor. It can be formulated as

$$h_i = \frac{(a_i P_{i,\max}^2 + b_i P_{i,\max} + c_i) + |e_i \sin(f_i (P_{i,\max} - P_{i,\max}))|}{(\alpha_i P_{i,\min}^2 + \beta_i P_{i,\min} + \gamma_i) + |\delta_i \sin(\eta_i (P_{i,\min} - P_{i,\min}))|} \quad [\$ / kg] \quad (18)$$

The average of the Min-Max, Max-Max, Min-Min, and Max-Min with valve point effect is called the average penalty factor. It can be formulated as

$$h_i = \frac{\left[\frac{(a_i P_{i,\min}^2 + b_i P_{i,\min} + c_i) + |e_i \sin(f_i (P_{i,\min} - P_{i,\min}))|}{(\alpha_i P_{i,\max}^2 + \beta_i P_{i,\max} + \gamma_i) + |\delta_i \sin(\eta_i (P_{i,\max} - P_{i,\max}))|} + \frac{(a_i P_{i,\max}^2 + b_i P_{i,\max} + c_i) + |e_i \sin(f_i (P_{i,\max} - P_{i,\max}))|}{(\alpha_i P_{i,\max}^2 + \beta_i P_{i,\max} + \gamma_i) + |\delta_i \sin(\eta_i (P_{i,\max} - P_{i,\max}))|} + \frac{(a_i P_{i,\min}^2 + b_i P_{i,\min} + c_i) + |e_i \sin(f_i (P_{i,\min} - P_{i,\min}))|}{(\alpha_i P_{i,\min}^2 + \beta_i P_{i,\min} + \gamma_i) + |\delta_i \sin(\eta_i (P_{i,\min} - P_{i,\min}))|} + \frac{(a_i P_{i,\max}^2 + b_i P_{i,\max} + c_i) + |e_i \sin(f_i (P_{i,\max} - P_{i,\max}))|}{(\alpha_i P_{i,\min}^2 + \beta_i P_{i,\min} + \gamma_i) + |\delta_i \sin(\eta_i (P_{i,\min} - P_{i,\min}))|} \right]}{4} \quad [\$ / kg] \quad (19)$$

Apart from the penalty factors discussed above, a common penalty factor exist for the multi-criteria dispatch problem. It can be formulated as follows

$$h_i = \frac{\sum_{i=1}^n \left[\frac{(a_i P_{i,\min}^2 + b_i P_{i,\min} + c_i) + |e_i \sin(f_i (P_{i,\min} - P_{i,\min}))|}{(\alpha_i P_{i,\max}^2 + \beta_i P_{i,\max} + \gamma_i) + |\delta_i \sin(\eta_i (P_{i,\max} - P_{i,\max}))|} + \frac{(a_i P_{i,\max}^2 + b_i P_{i,\max} + c_i) + |e_i \sin(f_i (P_{i,\max} - P_{i,\max}))|}{(\alpha_i P_{i,\max}^2 + \beta_i P_{i,\max} + \gamma_i) + |\delta_i \sin(\eta_i (P_{i,\max} - P_{i,\max}))|} + \frac{(a_i P_{i,\min}^2 + b_i P_{i,\min} + c_i) + |e_i \sin(f_i (P_{i,\min} - P_{i,\min}))|}{(\alpha_i P_{i,\min}^2 + \beta_i P_{i,\min} + \gamma_i) + |\delta_i \sin(\eta_i (P_{i,\min} - P_{i,\min}))|} + \frac{(a_i P_{i,\max}^2 + b_i P_{i,\max} + c_i) + |e_i \sin(f_i (P_{i,\max} - P_{i,\max}))|}{(\alpha_i P_{i,\min}^2 + \beta_i P_{i,\min} + \gamma_i) + |\delta_i \sin(\eta_i (P_{i,\min} - P_{i,\min}))|} \right]}{4n} \quad [\$ / kg] \quad (20)$$

G. Constraints of the Dispatch Problem

The combined economic dispatch problems with criterion functions (5) and (14) are solved for every type of the price penalty factors (6) to (11) and (15) to (20) under the constraints:

III. POWER BALANCE CONSTRAINT

$$\sum_{i=1}^n P_i = P_G = P_D \quad [MW] \quad (21)$$

where

P_G - Total power generation of the system

P_D - Total demand of the system

Generator operational limit constraints

$$P_{i,\min} \leq P_i \leq P_{i,\max}, i = \overline{1, n} \quad [MW] \quad (22)$$

where

$P_{i,\min}$ -Minimum value of real power allowed at generator

i

$P_{i,\max}$ -Maximum value of real power allowed at generator

i

IV. FORMULATION OF LAGRANGE'S METHOD FOR MULTI-CRITERIA DISPATCH PROBLEM

A. Multi-Criteria Dispatch Problem without Valve Point Effect Loading

The objective function in this case, is described by equation (5). The optimization problem is to find the optimal active power $P_i, i = \overline{1, n}$ produced by the generators in such a way that the criterion (5) is minimized and the constraints (21), (22) are satisfied. The problem has to be solved for every one of the price penalty factors (6) to (11). The problem is solved using Lagrange's method by introduction of the Lagrange's variable λ and formulation of a Lagrange's function:

$$L = F_T + \lambda \left(P_D - \sum_{i=1}^n P_i \right) \quad (23)$$

$$L = \sum_{i=1}^n \left[(a_i P_i^2 + b_i P_i + c_i) + h_i (\alpha_i P_i^2 + \beta_i P_i + \gamma_i) \right] + \lambda \left(P_D - \sum_{i=1}^n P_i \right) \quad (24)$$

The solution for the multi-criteria dispatch problem from equation (5), subject to the constraints (21) and (22) with penalty factors (6) or (7) or (8) or (9) or (10) or (11) is transferred to the problem for minimization of L according to $P_i, i = \overline{1, n}$, under the constraints (22).

Necessary conditions for optimality for solution of the problem (24), are:

$$\text{According to } P_i, \frac{\partial L}{\partial P_i} = 0, i = \overline{1, n} \quad (25)$$

$$\text{According to } \lambda, \frac{\partial L}{\partial \lambda} = 0 \quad (26)$$

Condition (25) is:

$$\frac{\partial L}{\partial P_i} = 2a_i P_i + b_i + 2h_i \alpha_i P_i + h_i \beta_i - \lambda = 0 \quad (27)$$

The real power P_i without valve point effect can be found from the equation (27) as follows:

$$P_i = \frac{\lambda - h_i \beta_i - b_i}{2(a_i + h_i \alpha_i)}, \quad i = \overline{1, n} \quad (28)$$

B. Multi-Criteria Dispatch Problem with Valve Point Effect Loading

The multi-criteria dispatch problem with valve point effect loading is formulated in the following way – find the optimal values of the active power $P_i, i = \overline{1, n}$ in such a way that the criterion function (14) is minimized for every type of the price penalty factors (15) to (20) under the constraints (21), (22). The Lagrange's function is determined in the same way as above

$$L = \left[\sum_{i=1}^n \left[(a_i P_i^2 + b_i P_i + c_i) + |e_i \sin(f_i (P_{i,\min} - P_i))| + h_i (\alpha_i P_i^2 + \beta_i P_i + \gamma_i) + |\delta_i \sin(\eta_i (P_{i,\min} - P_i))| \right] + \lambda \left(P_D - \sum_{i=1}^n P_i \right) \right] \quad (29)$$

The solution for the multi-criteria dispatch problem (29), subject to the constraints (21) and (22) with penalty factors (15) or (16) or (17) or (18) or (19) or (20) is transferred to the problem for minimization of L according to $P_i, i = \overline{1, n}$, under the constraints (22).

The necessary condition for optimality (25) has the following expression

$$\frac{\partial L}{\partial P_i} = \left[\begin{aligned} &2(a_i P_i + \alpha_i h_i P_i) + b_i + (h_i \beta_i) + \\ &+ \left| -e_i f_i \cos(f_i (P_{i,\min} - P_i)) \right| + \\ &+ \eta_i \delta_i h_i (\exp(\delta_i P_i)) - \lambda \end{aligned} \right] \quad (30)$$

Then the real power P_i with valve point effect can be calculated by an iteration procedure starting from a initial values of $P_i, i = \overline{1, n}$

$$P_i^{(k+1)} = \frac{\lambda - b_i - (h_i \beta_i) - \left| -e_i f_i \cos(f_i (P_{i,\min} - P_i^{(k)})) \right| - \eta_i \delta_i h_i (\exp(\delta_i P_i^{(k)}))}{2(c_i + \gamma_i h_i)} \quad (31)$$

If the value of the Lagrange's multiplier λ is known and some initial guess for the values of the active power P_i produced by the generators is available, the equation (31) can be solved to determine an updated value of the active power P_i with valve point effect.

C. Algorithm of the Lagrange's Method

The value of λ is unknown and has to be found from the necessary condition for optimality (26)

This condition is found from (24) to be:

$$\frac{\partial L}{\partial \lambda} = \left(P_D - \sum_{i=1}^n P_i = 0 = \Delta \lambda \right) \quad (32)$$

Equation (32) is not a function of λ , but represents the gradient of L according to λ . At the optimal solution this gradient has to be equal to zero. As analytical solution is not possible the gradient procedure for calculation of λ has to be developed as follows

$$\lambda^{(k+1)} = \lambda^k + \alpha \Delta \lambda^{(k)}, \quad \lambda \neq 0 \quad (33)$$

where

k is the iteration number and, $k = \overline{0 : m}$,

m is the given maximum number of iterations.

$\Delta \lambda^{(k)}$ is determined by the equation (32) and α is the step of the gradient procedure. The calculations start from some given initial value of the Lagrange's variable $\lambda^{(0)}$. When during the iterations $\Delta \lambda = 0$, the optimal solution for the Lagrange's variable is obtained. It will determine the optimal solution for the power that has to be produced by the generators.

The obtained solutions for $P_i, i = \overline{1, n}$ have to belong to the constraint domain (22). That is why for every step of the gradient procedure, the obtained solution is fit to the constraint domain following the procedure

$$P_i^{(k)} = \left\{ \begin{aligned} &P_{i,\min}, \quad \text{if } P_i^{(k)} < P_{i,\min} \\ &P_i^{(k)}, \quad \text{if } P_{i,\min} \leq P_i^{(k)} < P_{i,\max} \\ &P_{i,\max}, \quad \text{if } P_i^{(k)} > P_{i,\max} \end{aligned} \right\} \quad (34)$$

The condition for end of the iteration is

$$\Delta \lambda^{(k)} \leq \varepsilon \quad (35)$$

where $\varepsilon > 0$ is a small number.

The algorithm of the method is:

- 1) Initial value of the Lagrange's multiplier is guessed: $\lambda^{(0)}$, and the value of the condition for optimality ε is given.
- 2) Initial value of the active power $P_i^0, i = \overline{1, n}$ is guessed for the case of the dispatch problem with the valve point effect
- 3) Equation (28) or (31) is solved and $P^{(1)}$ is determined.
- 4) The obtained vector P_i is fit to the constraints (34).
- 5) $\Delta \lambda^{(0)}$ is calculated using equation (32) where $P^{(1)}$ is substituted.
- 6) The condition (35) is checked. If it is fulfilled the calculations stop, if not, improved value of $\lambda \rightarrow \lambda^{(1)}$ is calculated using equation (33)
- 7) Calculations of the improved values of $P_i \rightarrow P_i^{(2)}$ is done as using equations (28) or (31) with or without valve point effect. Iterations continue until condition (35) is satisfied until the maximum number of iterations m is reached.
- 8) The optimal solution is used to calculate the total cost, using equations (5), and (14).

The algorithm of the method is shown in Fig. 1.

V. SIMULATION RESULTS OF MULTI-CRITERIA DISPATCH PROBLEM

The five generator system is represented by fuel cost and emission coefficients with valve point effect and real power limits as given in Table I. The problem is formulated as a Multi-objective function using the various penalty factors. The dispatch problem is solved in Matlab environment. The Lagrange's method is used to obtain the solution of the dispatch problem. The solution of the multi-criteria dispatch problem without valve point effect loading is shown in Table II to Table VII for the considered penalty factors (6) to (11). The comparison of the dispatch problem solution for the various penalty factors without valve point effect is shown in Table VIII.

The comparison is done for the maximum power demand of $P_D = 400$ [MW]. Since the bigger variations in the dispatch problem solution exists for maximum power demand. The results show that the fuel cost and emission values are the same for various penalty factors without valve point effect. The Min-Max penalty factor provides minimum fuel cost of combined dispatch problem in compare to other penalty factors. The solution of the multi-criteria dispatch problem with valve point effect loading is shown in Table IX to Table XIV for various penalty factors. The comparison of the dispatch problem solution for various penalty factors with valve point effect is shown in Table XV. The results show that Min-Max price penalty factor is good to yield the minimum fuel cost, combined dispatch problem fuel cost, and Computation Time (CT) values of various penalty factors with valve point effect loading. Comparison of the solution with and without valve point effect for the considered penalty factors is shown in Table XVI. The comparison is done on the basis of the Min-Max

solution of the dispatch problem with valve point effect of the power demand P_D 400 [MW]. The comparison results show that, there will be smaller variations in the values of the fuel cost, emission, combined fuel cost and computation time in comparison to the solution without valve point effect.

Fig. 1. Flow diagram of multi-criteria dispatch problem with valve point effect using Lagrange's method

TABLE I: DATA OF THE MULTI-CRITERIA DISPATCH PROBLEM DATA OF FIVE GENERATOR SYSTEM

Generator		1	2	3	4	5
Fuel cost coefficients and valve point effect coefficients	a_i	25	60	100	120	40
	b_i	2.0	1.8	2.1	2.0	1.8
	c_i	0.0080	0.0030	0.0012	0.0010	0.0015
	e_i	100	140	160	180	200
	f_i	0.0042	0.040	0.038	0.037	0.035
Emission coefficients and valve point effect coefficients	α_i	80	50	60	45	30
	β_i	-0.805	-0.555	-1.355	-0.600	-0.555
	γ_i	0.0180	0.0150	0.0105	0.0080	0.0120
	δ_i	0.6550	0.5773	0.4968	0.4860	0.5035
	η_i	0.02846	0.02446	0.02270	0.01948	0.02075
Generator real power operational limits	$P_{i, \min}$	10	20	30	40	50
	$P_{i, \max}$	75	125	175	250	300

TABLE II: MULTI-CRITERIA DISPATCH PROBLEM SOLUTION USING MIN-MAX PRICE PENALTY FACTOR WITHOUT VALVE POINT EFFECT

P _D	lambda	P1	P2	P3	P4	P5	F _C	E _T	F _T	CT
200	38.633	36.633	36.833	36.534	40.000	50.000	540807.100	402124.600	555113.266	3.428
250	51.940	49.940	50.140	49.840	49.940	50.140	861920.600	661720.000	884665.550	3.530
300	61.940	59.940	60.140	59.840	59.940	60.140	1241304.000	953063.200	1274065.637	3.331
350	71.940	69.940	70.140	69.840	69.940	70.140	1689687.000	1297406.000	1734288.392	3.176
400	81.940	79.940	80.140	79.840	79.940	80.140	2207070.000	1694750.000	2265333.814	3.064

TABLE III: MULTI-CRITERIA DISPATCH PROBLEM SOLUTION USING MAX-MAX PRICE PENALTY FACTOR WITHOUT VALVE POINT EFFECT

P _D	lambda	P1	P2	P3	P4	P5	F _C	E _T	F _T	CT
200	38.630	36.632	36.833	36.535	40.000	50.000	540815.100	402123.700	1081171.254	3.482
250	51.936	49.939	50.139	49.842	49.940	50.141	861927.400	661716.800	1723337.987	3.920
300	61.936	59.939	60.139	59.842	59.940	60.141	1241312.000	953059.500	2482062.049	3.186
350	71.936	69.939	70.139	69.842	69.940	70.141	1689696.000	1297402.000	3378805.616	3.467
400	81.936	79.939	80.139	79.842	79.940	80.141	2207081.000	1694745.000	4413568.689	3.348

TABLE IV: MULTI-CRITERIA DISPATCH PROBLEM SOLUTION USING MIN-MIN PRICE PENALTY FACTOR WITHOUT VALVE POINT EFFECT

P _D	lambda	P1	P2	P3	P4	P5	F _C	E _T	f _t	CT
200	38.630	36.632	36.833	36.535	40.000	50.000	540815.100	402123.700	1081961.310	3.118
250	51.936	49.939	50.139	49.842	49.940	50.141	861927.400	661716.900	1724679.537	3.122
300	61.936	59.939	60.139	59.842	59.940	60.141	1241312.000	953059.500	2483994.351	3.131
350	71.936	69.939	70.139	69.842	69.940	70.141	1689696.000	1297402.000	3381436.153	3.224
400	81.936	79.939	80.139	79.842	79.940	80.141	2207081.000	1694745.000	4417004.942	2.990

TABLE V: MULTI-CRITERIA DISPATCH PROBLEM SOLUTION USING MAX-MIN PRICE PENALTY FACTOR WITHOUT VALVE POINT EFFECT

P _D	lambda	P1	P2	P3	P4	P5	F _C	E _T	F _T	CT
200	38.626	36.631	36.832	36.537	40.000	50.000	540823.200	402122.700	21257001.400	3.361
250	51.933	49.938	50.138	49.844	49.939	50.142	861934.300	661713.700	34040621.700	3.336
300	61.933	59.938	60.138	59.844	59.939	60.142	1241320.000	953055.700	49030097.010	2.958
350	71.933	69.938	70.138	69.844	69.939	70.142	1689706.000	1297398.000	66746702.440	3.227
400	81.933	79.938	80.138	79.844	79.939	80.142	2207092.000	1694740.000	87190438.000	3.069

TABLE VI: MULTI-CRITERIA DISPATCH PROBLEM SOLUTION USING AVERAGE PRICE PENALTY FACTOR WITHOUT VALVE POINT EFFECT

P _D	lambda	P1	P2	P3	P4	P5	F _C	E _T	F _T	CT
200	38.627	36.631	36.832	36.537	40.000	50.000	540821.600	402123.000	5993811.195	3.562
250	51.933	49.938	50.138	49.843	49.939	50.142	861932.800	661714.400	9593327.149	3.020
300	61.933	59.938	60.138	59.843	59.939	60.142	1241318.000	953056.600	13817555.910	3.346
350	71.933	69.938	70.138	69.843	69.939	70.142	1689704.000	1297399.000	18810309.480	3.235
400	81.933	79.938	80.138	79.843	79.939	80.142	2207090.000	1694741.000	24571587.880	3.208

TABLE VII: MULTI-CRITERIA DISPATCH PROBLEM SOLUTION USING COMMON PRICE PENALTY FACTOR WITHOUT VALVE POINT EFFECT

P _D	lambda	P1	P2	P3	P4	P5	F _C	E _T	F _T	CT
200	38.628	36.632	36.832	36.536	40.000	50.000	540816.200	402123.900	1631406.497	3.190
250	51.935	49.939	50.139	49.842	49.939	50.141	861927.900	661716.800	2608201.405	3.236
300	61.935	59.939	60.139	59.842	59.939	60.141	1241312.000	953059.400	3756553.552	3.297
350	71.935	69.939	70.139	69.842	69.939	70.141	1689697.000	1297402.000	5113810.663	3.312
400	81.935	79.939	80.139	79.842	79.939	80.141	2207082.000	1694745.000	6679972.739	3.130

TABLE VIII: COMPARISON OF MULTI-CRITERIA DISPATCH PROBLEM SOLUTION USING VARIOUS PRICE PENALTY FACTOR WITHOUT VALVE POINT EFFECT LOADING

Criterion	Impact of various penalty factors of multi-criteria dispatch problem solution without valve point effect loading when P _D =400 [MW]					
	Min-Max	Max-Max	Min-Min	Max-Min	Average	Common
F _C	100 %	100 %	100 %	100 %	100 %	100 %
E _T	100 %	99.99%	99.99%	99.99%	100 %	100 %
F _T	100 %	194.83%	194.98%	3848.90%	1084.67%	294.87%
CT	100 %	109.26%	97.58%	100.16%	104.70%	102.15%

TABLE IX: MULTI-CRITERIA DISPATCH PROBLEM SOLUTION USING MIN-MAX PRICE PENALTY FACTOR WITH VALVE POINT EFFECT LOADING

P _D	lambda	P1	P2	P3	P4	P5	F _{CVP}	E _{TVP}	F _{TVP}	CT
200	42.559	39.135	36.329	34.536	40.000	50.000	529342.332	406934.633	543259.496	3.305
250	55.332	52.452	51.928	48.689	46.931	50.000	832799.182	671089.805	854695.480	3.469
300	65.390	61.108	62.816	61.038	58.230	56.809	1239748.925	968739.646	1272452.037	3.164
350	74.876	69.516	70.634	72.560	69.943	67.347	1716417.407	1307904.172	1761812.973	3.463
400	84.670	78.615	78.924	80.551	82.660	79.249	2249640.856	1690782.844	2309050.367	3.644

TABLE X: MULTI-CRITERIA DISPATCH PROBLEM SOLUTION USING MAX-MAX PRICE PENALTY FACTOR WITH VALVE POINT EFFECT LOADING

P _D	lambda	P1	P2	P3	P4	P5	F _{CVP}	E _{TVP}	F _{TVP}	CT
200	41.577	38.254	35.943	35.803	40.000	50.000	534878.984	405432.080	1069099.537	3.543
250	54.122	51.631	51.215	49.850	47.304	50.000	841917.474	669022.407	1682947.109	3.133
300	64.209	60.296	62.380	62.327	58.677	56.320	1253988.402	968360.420	2506900.424	3.139
350	73.798	68.766	70.277	73.465	70.561	66.931	1732214.801	1307253.243	3463235.175	3.201
400	83.659	77.886	78.612	81.523	83.100	78.880	2265987.107	1690162.676	4530767.379	2.975

TABLE XI: MULTI-CRITERIA DISPATCH PROBLEM SOLUTION USING MIN-MIN PRICE PENALTY FACTOR WITH VALVE POINT EFFECT LOADING

P _D	lambda	P1	P2	P3	P4	P5	F _{CVP}	E _{TVP}	F _{TVP}	CT
200	41.575	38.254	35.942	35.803	40.000	50.000	534881.255	405432.799	1069904.824	4.199
250	54.120	51.632	51.214	49.851	47.303	50.000	841917.450	669024.799	1684313.711	3.290
300	64.208	60.296	62.380	62.328	58.677	56.319	1253992.305	968366.430	2508883.901	3.037
350	73.796	68.767	70.277	73.466	70.560	66.930	1732216.638	1307259.655	3465890.285	4.192
400	83.657	77.887	78.611	81.524	83.099	78.880	2265990.399	1690169.675	4534185.116	2.987

TABLE XII: MULTI-CRITERIA DISPATCH PROBLEM SOLUTION USING MAX-MIN PRICE PENALTY FACTOR WITH VALVE POINT EFFECT LOADING

P _D	lambda	P1	P2	P3	P4	P5	F _{CVP}	E _{TVP}	F _{TVP}	CT
200	-4.418	10.000	20.000	70.112	49.888	50.000	917344.439	509774.132	34000867.870	3.099
250	10.832	19.116	29.902	83.031	67.952	50.000	1407173.883	770167.185	52741433.690	3.258
300	22.160	31.777	43.004	92.580	82.639	50.000	1913887.634	1069586.977	72477586.370	3.090
350	33.804	45.680	57.685	102.947	92.020	51.669	2435689.054	1430045.275	92813282.970	4.119
400	44.612	55.406	67.119	113.346	100.930	63.199	3015215.214	1819592.900	115169251.900	3.279

TABLE XIII: MULTI-CRITERIA DISPATCH PROBLEM SOLUTION USING AVERAGE PRICE PENALTY FACTOR WITH VALVE POINT EFFECT LOADING

P _D	lambda	P1	P2	P3	P4	P5	F _{CVP}	E _{TVP}	F _{TVP}	CT
200	32.440	31.514	32.581	45.683	40.222	50.000	591979.226	405393.643	6436098.261	3.536
250	43.232	44.371	45.312	58.698	51.620	50.000	937620.470	661601.586	10265336.050	3.088
300	53.600	54.105	58.466	71.774	63.802	51.853	1390591.190	977804.790	15254576.340	3.202
350	63.855	62.972	67.162	80.135	76.949	62.782	1881452.548	1312504.047	20702177.490	3.317
400	74.382	72.410	75.961	88.894	87.570	75.164	2415116.064	1696370.162	26618980.110	3.217

TABLE XIV: MULTI-CRITERIA DISPATCH PROBLEM SOLUTION USING COMMON PRICE PENALTY FACTOR WITH VALVE POINT EFFECT LOADING

P _D	lambda	P1	P2	P3	P4	P5	F _{CVP}	E _{TVP}	F _{TVP}	CT
200	40.605	37.670	35.634	36.696	40.000	50.000	538920.898	404663.250	1626562.659	3.252
250	52.918	51.017	50.580	50.580	47.824	50.000	849728.563	667366.296	2570728.181	3.222
300	63.052	59.710	62.009	63.170	59.309	55.803	1266658.237	968384.780	3825134.854	3.404
350	72.707	68.219	69.955	74.055	71.331	66.442	1746822.719	1307167.892	5269826.767	4.107
400	82.646	77.382	78.345	82.185	83.642	78.446	2280483.270	1690340.038	6886389.421	3.118

TABLE XV: COMPARISON OF MULTI-CRITERIA DISPATCH PROBLEM SOLUTION USING VARIOUS PRICE PENALTY FACTOR WITH VALVE POINT EFFECT LOADING

Criterion	Impact of various penalty factors of multi-criteria dispatch problem solution with valve point effect loading					
	Min-Max	Max-Max	Min-Min	Max-Min	Average	Common
F _{CVP}	100 %	100.72%	100.72%	134.03%	107.35%	101.37%
E _{TVP}	100 %	99.96%	99.96%	107.61%	100.33%	99.97%
F _{TVP}	100 %	196.21%	196.36%	4987.73%	1152.81%	298.23%
CT _{TVP}	100 %	81.64%	81.97%	89.96%	88.28%	85.56%

TABLE XVI: COMPARISON OF MULTI-CRITERIA DISPATCH PROBLEM SOLUTION USING VARIOUS PRICE PENALTY FACTOR FOR THE CASES WITH AND WITHOUT VALVE POINT EFFECT LOADING

Criterion comparison	Min-Max	Max-Max	Min-Min	Max-Min	Average	Common
F _C Vs F _{CVP}	0%	0.72%	0.72%	34.03%	7.35%	1.37%
E _T Vs E _{TVP}	0%	-0.03%	-0.03%	7.62%	0.33%	0.03%
F _T Vs F _{TVP}	0%	0.62%	1.38%	1138.83%	68.14%	3.36%
CT Vs CT _{TVP}	0%	-27.62%	-15.61%	-11.20	-16.5%	-16.59%

VI. CONCLUSION

The multi-criteria dispatch problem of both fuel cost function and emission function are formulated without and with valve point effect loading. The Lagrange's algorithm is developed for the solution of the dispatch problem. The five generator multi-objective dispatch problem is solved separately with and without valve point effect for various price penalty factors such as i)Min-Max ii) Max-Max, iii)Min-Min, iv)Max-Min, v)Average, and v) Common. It concludes that Min-Max price penalty factor is good to yield the minimum fuel cost, and combined dispatch problem fuel cost values in comparison to other penalty factors. The comparison results of the solution of the dispatch problem with and without valve point effect show that, there exist smaller variations in fuel cost, emission, combined fuel cost, and computation time.

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