An Indirect Adaptive Predictive Control with Augmented UD Identifier for Linear Time Varying Systems

Pouria Sarhadi, Karim Salahshoor, and Ali Khaki-Sedigh

Abstract—In this paper, an indirect adaptive generalized predictive controller (GPC) is proposed by incorporating an augmented UD identifier (AUDI), based on Bierman's UD factorization algorithm. The developed adaptive control scheme is mainly aimed to deal with systems having linear time varying (LTV) dynamic characteristics. A series of simulation studies has been conducted to reveal the effectiveness of the developed adaptive control scheme to cope with such time varying dynamic profiles. The obtained results illustrate the controller robustness against both external disturbances and parameters uncertainties.

Index Terms—Indirect adaptive control, predictive control, AUDI, linear time varying system

I. INTRODUCTION

Dealing with the Time Varying (TV) dynamic is one of the interesting problems in control theory. Simplicity of the linear control theory is often made the designers to linearize nonlinear systems so as to control them with powerful theory of linear systems [1]. This produces a series of systems called as Linear Time Varying (LTV) Systems. In this paper, a suitable adaptive controller is proposed for this application purpose.

Gain Scheduling (GS) offers a simple control technique to handle LTV systems whenever the variation of parameters are certain or predictable by preset [1]-[3]. It fits a sequence of controller gains to different system operating points through tracking a measured indicating parameter of the system. However, variation of system dynamics is often unknown to find an appropriate scheduling variable to be monitored, leading to the need for designing a robust or adaptive control scheme. In some cases, there might be sufficient information about system model parameters and hence a robust control technique with no online identification is adequate [4]. But, where the variation range of model parameters is wide and unknown, adaptive control scheme is the only suitable candidate.

Predictive control is one of the advanced control methodologies which have made a significant impact on industrial control engineering [5]. This controller is attracted

an increasingly attention in control literature in this decade [5]-[8]. Perhaps, GPC can be regarded as the most famous controller in the predictive controllers. Adaptive versions of GPC have been used in several researches. In [9], it was used for controlling a flight control system. In [10], another version of adaptive GPC has been utilized in a solar plant. This controller has been used in a microbial process plant [11] and [12] employed it on a water distribution canal. In [13], another kind of this controller has been implemented on a smart structure. These applications show the practical effectiveness of this controller.

This paper presents a combination of AUDI and GPC scheme to develop an adaptive GPC controller for LTV systems (Fig. 1). In conventional adaptive controllers, various kinds of Recursive Least Squares (RLS) algorithm have been used [1], [14]. In all these configurations, plant parameters are first identified by RLS and then the so called certainly equivalence principle is invoked to compute the control signal sequence. But, RLS is known to have a number of shortcomings such as poor numerical performance, instability conditions and etc. As a consequence, an Augmented UD Identifier (AUDI) is utilized in this work to boost up the estimation robustness. Augmented UD Identification method has already been known [15]-[17], being developed on the basis of Bierman's UD Factorization algorithm to provide more numerical robustness.

Fig. 1. Indirect adaptive predictive controller structure.

II. AUGMENTED UD IDENTIFICATION ALGORITHM

Robustness of parameter estimator plays a key role in adaptive controllers. In practical cases, RLS algorithm often leads to negative definite covariance matrix, making the estimation unstable. To prevent this problem, UD factorization method was proposed [18]. It uses the factored covariance matrix and hence updates the factors instead of the full matrix to guarantee the positive definiteness of covariance matrix. Niu et. Al proposed a better method by rearranging the data vectors and augmenting the current input to it. This method, called as AUDI, incorporates the following nice characteristic features [15]:

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- 1) It simultaneously estimates model order and relevant order's parameters;
- 2) It is inherently a least squares algorithm;
- 3) It has good numerical properties and stability;
- 4) It is much easier to interpret and understand than the original UD algorithm;

AUDI gives these properties with the same computation burden of an n th-order RLS algorithm. For appropriate tracking of time varying model parameters, it is decided to employ a forgetting factor in the AUDI algorithm.

Consider a system with the following input-output Auto Regressive with eXogenous input (ARX) model structure:

$$A(q^{-1})y(t) = B(q^{-1})u(t-1) + e(t) \quad (1)$$

$y(t)$, $u(t)$ and $e(t)$ indicate output, input of the system and the modeling error respectively as the zero mean white noise. q^{-1} denotes time backward shift operator, $A(q^{-1})$ and $B(q^{-1})$ represent input-output digital transfer functions denominator and numerator coefficients as:

$$A(q^{-1}) = 1 + a_1 q^{-1} + a_2 q^{-2} + \dots + a_{n_a} q^{-n_a}$$

$$B(q^{-1}) = b_0 q^{-1} + b_1 q^{-2} + \dots + b_{n_b} q^{-n_b}$$

First the input-output data should be filtered as:

$$y_f(t) = H_f(q^{-1})y(t), \quad u_f(t) = H_f(q^{-1})u(t)$$

$$H_f(q^{-1}) = \frac{(1-\beta)(1-q^{-1})}{(1-\beta q^{-1})^2}$$

where f subscript introduces the filtered data. The filtered input-output data could provide a suitable data set for estimator. The filter may be chosen as a band-pass filter. This type of filter selection enforces the parameter estimator to focus on the relevant frequency band thereby significantly reduces the bad effects of unmodeled dynamics and sensor noise in high frequency and meanwhile attenuates load disturbances in low frequency [19]. The resulting estimation algorithm can be described as follows:

Initialization: Set $U(0)D(0)U^T(0) = P_0 I$

where D and U are diagonal and unit upper triangular matrixes with P_0 chosen as a large number.

- 1) Construct the data vector:
- 2) $\varphi_j(t) = [-y_f(t-n) - u_f(t-n) - y_f(t-n-1) \dots - y_f(t-n_{j-1})]$
2. Compute:
- 3) For $j = 1, \dots, 2n_a + 1$ go through steps 4-6
- 4) Compute:

$$\beta_j = \beta_{j-1} + f_j g_j$$

$$D(t)_{jj} = \frac{\beta_{j-1} D(t-1)_{jj}}{\beta_j \lambda(t)}$$

$$v_j = g_j$$

$$\mu_j = -\frac{f_j}{\beta_{j-1}}$$

λ has the same role of forgetting factor in RLS Identifier.

- 5) For $i = 1, \dots, j-1$, do step 6 if $j = 1$ skip it.
- 6) Compute:

$$U(t)_{ij} = U(t-1)_{ij} + v_i \mu_j$$

$$v_i := v_i + U(t-1)_{ij} v_j$$

The estimated parameters are then extracted as:

$$\hat{\theta}(t) = U(1:2n, \text{end})$$

$$\hat{\theta}(t) = [a_{n_a}, b_{n_b}, \dots, a_2, b_2, a_1, b_1]^T$$

In this algorithm, D and U matrixes have meaningful data. For instance, $U(t)$ contains model parameters from 1 to n as follows:

$$U(t) = \begin{bmatrix} 1 & \hat{\alpha}_0(t-n) & & & & \\ 0 & 1 & \hat{\theta}_1(t-n+1) & \hat{\alpha}_1(t-n+1) & & \\ & \ddots & 1 & & \hat{\theta}_{n-1}(t-1) & \\ & & & 1 & & \hat{\alpha}_{n-1}(t-1) & \hat{\theta}_n(t) \\ \vdots & & & 0 & \ddots & & \\ & & & & 1 & & \\ & & & & \ddots & & \\ 0 & \dots & & & & 1 & 0 & 1 \end{bmatrix}$$

whereas, $D(t)$ contains loss functions for the equivalent models which can be represented by:

$$D^{-1}(t) = \text{diag}[J_0(t-n), L_0(t-n), \dots, J_{n-1}(t-1), L_{n-1}(t-1), J_n(t)]$$

With:

$$J_n(t) = \sum_{k=1}^t (y^2(k) - \hat{\theta}_n^T(t) P_n^{-1}(t) \hat{\theta}_n(t)) = \sum_{k=1}^t e^2(k)$$

$$P_n(t) = \sum_{k=1}^t (h_n^T(k) h_n(k))$$

$$\text{where } h_n^T(k) = [-y(t-n), -u(t-n), \dots, -y(t-1), u(t-1)]$$

III. GENERALIZED PREDICTIVE CONTROL

GPC was introduced by Clarke et. al. at 1987 [20] and it is now regarded as one of the most popular and practical controllers. In this paper, the GPC in its two degree of freedom formulation is used [21]. This form of GPC has been shown in Fig. 2.

$d(t)$, $v(t)$ and $n(t)$ are output, input disturbance and sensor noise, respectively. Consider the CARIMA (Controlled Auto Regressive and Moving Average) model:

$$\Delta(q^{-1})y(t) = B(q^{-1})u_c(t-1) + T(q^{-1})e(t)/\Delta \quad (1)$$

where $\Delta = 1 - q^{-1}$ is difference operator, and $T(q^{-1})$ is a user specified observer or prefilter polynomial as:

$$T(q^{-1}) = 1 + c_1 q^{-1} + c_2 q^{-2} + \dots + b_{n_t} q^{-n_t}$$

The choice of T polynomial has raised several alternative discussions [21]-[22]. But, it is clearly known that the correct selection of this polynomial could enhance the resulting closed loop robustness against the inevitable unmodeled dynamics with no excessive control actions. On the other hand, T is treated as a design parameter that can influence the robust stability of GPC [6].

The basic idea of GPC is to calculate a sequence of future control signals, leading to minimization of a multistage cost

function defined over a prediction horizon [6]. This function can be represented by:

$$J = \sum_{j=N_1}^{N_2} [\hat{y}(t+j|t) - r(t+j)]^2 + \sum_{j=1}^{N_u} \rho(j) [\Delta u(t+j-1)]^2$$

where $\hat{y}(t+j|t)$ denotes an optimum j -step ahead prediction of the system output data from maximum prediction horizon to t , N_2 and N_1 represent the maximum and minimum output horizons, N_u is the control horizon, $\rho(j)$ is weighting sequence, and $r(t)$ is the future set point trajectory.

Fig. 2. Two degree of freedom structure of GPC

In this GPC form, two Diophantine equations are utilized as (Notice that the argument q^{-1} will be dropped when no confusion arises):

$$T = E_j A \Delta + q^{-j} F_j$$

$$\deg(E_j) = j - 1$$

$$\deg(F_j) = \max(\deg(A), \deg(T) - j)$$

$$E_j B = H_j T + q^{-j} N_j$$

$$\deg(H_j) = j - 1$$

$$\deg(N_j) = \max(\deg(B), \deg(T)) - 1$$

Then, the optimal prediction of output in j -th step ahead can be determined with some manipulations:

$$\hat{y}(t+j|t) = H_j \Delta u(t+j-1) + \frac{N_j}{T} \Delta u(t-1) + \frac{F_j}{T} y(t)$$

$$f(t+j) = \frac{N_j}{T} \Delta u(t-1) + \frac{F_j}{T} y(t)$$

where f indicates the free response, including past and now, output and control signal sequence. By minimizing the cost function, the control signal is calculated as:

$$\Delta u(t) = K_{GPC} (r - f)$$

$$r = [r(t+N_1) \ r(t+N_1+1) \ \cdots \ r(t+N_2)]^T$$

Noting that the control signal is taken with respect to future errors, not past errors, as is the usual case in the conventional feedback controllers. Where:

As a result, the control signal is determined as:

$$K_{GPC} = [K_{N_1}, \dots, K_{N_2}] = [100 \cdots 0] (G^T M G + \Lambda)^{-1} G^T M$$

$$G = \begin{bmatrix} h_0 & 0 & \cdots & 0 \\ h_1 & h_0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ h_{N_2-1} & h_{N_2-2} & \cdots & h_0 \end{bmatrix}_{N_2 \times N_2}$$

$$\Delta u(t) = K_{GPC} (r - f) = \sum_{j=N_1}^{N_2} k_j [r(t+j) - f(t+j)] =$$

$$= \sum_{j=N_1}^{N_2} k_j r(t+j) - \sum_{j=N_1}^{N_2} k_j \frac{N_j}{T} \Delta u(t-1) - \sum_{j=N_1}^{N_2} k_j \frac{F_j}{T} y(t)$$

In this equation, it has been assumed that the future reference trajectory keeps constant along the horizon. It has been observed that in complicated systems the use of future set points adds unstable zeros to the closed loop transfer function or causes phase-lead even at high frequencies [22]. With some manipulations, it can be rewritten:

$$\Delta u(t) = \frac{T(q^{-1})}{R(q^{-1})} w(t) - \frac{S(q^{-1})}{R(q^{-1})} y(t)$$

$$u_c(t) = \Delta u(t) + u_c(t-1)$$

With:

$$R(q^{-1}) = \frac{\left[T(q^{-1}) + q^{-1} \sum_{j=N_1}^{N_2} k_j N_j(q^{-1}) \right]}{\sum_{j=N_1}^{N_2} k_j}$$

$$S(q^{-1}) = \frac{\sum_{j=N_1}^{N_2} k_j F_j(q^{-1})}{\sum_{j=N_1}^{N_2} k_j}$$

The degrees of these polynomials are:

$$\deg(R) = \deg(N_j) + 1$$

$$\deg(S) = \max(\deg(A), \deg(T))$$

Substituting the calculated control signal into the closed loop relationship, leads to:

$$y(t) = \frac{B T q^{-1}}{A R \Delta + B S q^{-1}} r(t) + \frac{T R}{A R \Delta + B S q^{-1}} e(t)$$

IV. SIMULATION RESULTS

In this section, the developed adaptive controller is tested on a LTV system. The employed model is, in fact, the linearized model of a highly nonlinear fermentation process [23] discretized with sampling time $T_s = 720$ sec, leading to the following digital transfer function:

$$G(q^{-1}) = \frac{b_{1d}(t) + b_{2d}(t)q^{-1}}{1 + a_{1d}(t)q^{-1} + a_{2d}(t)q^{-2}}$$

As shown, the model parameters are prone to varying under various operational conditions. A typical dynamic variation profile has been shown in Fig. 4. But, it is assumed that the designer has no a priori information about the dynamic system variation profile so as to adopt a scheduling strategy.

The AUDI tuning parameters are set to:

$$\hat{\theta}(0) = 0, P_0 = 10^9, \lambda = 0.98$$

Furthermore, the GPC parameters are set to:

$$\rho = 0.8, T = (1 - 0.6)^2$$

$$N_1 = N_u = 3, N_2 = 5$$

First, the developed controller is evaluated under the nominal situation with no external perturbation. Simulation results are shown in Fig. 3. The true and estimated parameters are shown in Fig. 4. The results indicate that the parameter identifier is able to track the varying system parameters in such a weak excitation condition.

Fig. 3. Output of the system (upper figure) and control signal (lower figure) for the system nominal condition

Fig. 4. True and estimated parameters for nominal simulation

Fig. 5. Output of the system (upper figure) and control signal (lower figure) with added noise and disturbances

In second simulation study, external perturbations are induced to excite the system. The output noise n is assumed to be a zero mean white noise with standard deviation $\sigma = 0.01$. A step output disturbance d with amplitude 0.5 is added to the system output in the

interval $24 < t < 64$. In addition, a similar input disturbance v is added in the interval $50 < t < 95$. Fig. 5 shows the quality of designed controller in the presence of various perturbations. As illustrated, the external disturbances have been eliminated and the induced noise effects have been attenuated with a reasonable control signal.

In third simulation study, the control system has been faced with a more severe condition. A further gain uncertainty of 2 and 0.5 has been introduced in the second simulation test condition. This has been implemented with multiplying the control signal or nominator coefficients to the uncertain gain. It is expected that controller could adapt itself with this situation. Fig. 6 shows the results for both uncertain gains. The controller has tolerated the added perturbations with gain uncertainty simultaneously, and shows a very good robust performance. The superior performance of the estimator is clearly represented in Fig. 7. The induced gain uncertainty has been recognized by the estimator and hence the AUDI is able to track the variation of system nominator transfer function coefficients.

Fig. 6. Output of the system (upper figure) and control signal (lower figure) with added noise and disturbances for gain uncertainties ($K=2$, $K=0.5$)

Fig. 7. True and estimated parameters for perturbed condition with gain uncertainty ($K=2$, $K=0.5$)

V. CONCLUSIONS

An adaptive predictive control scheme with augmented UD identifier has been developed to handle systems with linear time varying dynamic model. The simulation results have illustrated the excellent performance of this controller in various operating conditions. The simulation studies were organized to cover probable variations in both dynamic parameters and perturbations. The robustness of controller was clearly validated in all the perturbation conditions, leading to its promising capabilities to be considered in real practical applications.

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