Multidimensional Fast Multiplierless DCT Algorithm Using Ramanujan Ordered Numbers

Geetha K. S. and M. Uttarakumari

Abstract—This paper proposes a new multidimensional (m -D) multiplierless Ramanujan Numbered Discrete Cosine Transform (RDCT) based coding to improvise the coding efficiency in the visual data compression for multimedia applications. In this paper, the m D-DCT with size $N_1 \times N_2 \times \dots \times N_m$, where N_i power of 2 is, can be converted into a series of one dimensional (1-D) DCT's which are evaluated using Ramanujan Ordered Numbers thereby making the algorithm multiplierless. The proposed algorithm achieves considerable savings in terms of shift operations compared with the row-column approach thus making it suitable for image and video coding applications. For a 3-dimensional input data of size $8 \times 8 \times 8$, the algorithm reduces the number of shift operations by 66% compared to multidimensional row-column approach and approximately 0.1% increase in the number of adders compared with the row-column approach. Compared with the other recently presented multidimensional DCT algorithms, the proposed one gives better result as it uses the shift operations instead of the floating-point multipliers and also achieves considerable reduction in the number of addition operation.

Index Terms—Discrete cosine transform, fast multiplierless algorithm, Ramanujan ordered number, polynomial transform

I. INTRODUCTION

The Discrete Cosine Transform (DCT) approaches the statistically optimal Karhunen-Loeve transform (KLT) for highly correlated data, so it is widely used in signal processing mainly for image and video compression [1], [2]. All existing digital media compression standards-JPEG, MPEG, and H.26x- are mainly based on DCT. The m -D DCT is chosen as an alternate approach for many image and video applications. For example, the 3-D DCT coding performs better than the motion compensated transform coding technique used in video coding standards [3]. The multidimensional transforms are also used in different areas of computer vision, HDTV video coding, and multi-frame detection [4].

Recently, many multidimensional transform algorithms have been proposed [5]-[8] which are reported to be less computationally complex compared with the conventional row-column approach [6]. In [9],[10] a recursive algorithm for generating higher order m -D DCT by combining the computation of 2^m identical lower order DCT architectures is presented, but it requires too many operations. The algorithm proposed in [11] converts the m -D DCT with size

$N \times N \times \dots \times N$ (N is a power of 2) into 1-D DCT's and greatly reduces the number of operations, but it cannot support applications that require different dimensional sizes. The m -D algorithm in [12] is based on the factorization of the tensor product form of m -D DCT, uses $1/m$ times the number of multiplications as required by the conventional row-column approach and considerable less number of addition operations compared with the row-column approach. In this method, the m -D data is decomposed into 1-D independent vectors based on the factorization of the tensor product, but the algorithm requires additional shifts and addition operation in both pre-processing and post-processing stages. Many polynomial transform (PT)-based algorithms for 2-D DCT [13],[14] and algorithms for multidimensional DCT [15],[16] are reported to offer reductions on the required number of operations, but these algorithms are based on the relationship between DCT and DFT and then expressing the DFT as a polynomial evaluation by polynomial transformation. The fast algorithm given by Zeng [17] can reduce the overall computational and structural complexity. The algorithm uses a technique to directly decompose the multidimensional DCT into 1-D DCT's in order to avoid operations on the complex data, but requires the floating-point multiplication operations to compute the 1-D DCT's, thus being not VLSI friendly. In [18] a new PT-based algorithm is proposed to directly decompose the 2-D DCT into 1-D DCT's so that operations on the complex data is not required. This algorithm uses Ramanujan ordered numbers to compute DCT's thereby replacing floating-point multipliers using shifters and adders. This method also does not require any post-addition as reported in [6] which makes the algorithm to be simpler for implementation.

In this paper, a generalized multidimensional multiplierless algorithm is proposed. The m -D data whose size is $N_1 \times N_2 \times \dots \times N_m$ where N_i ($i = 1, 2, \dots, m$) is power of 2, are directly decomposed into 1-D data. The 1-D DCT's are then computed by evaluating the cosine angles of the multiples of $2\pi/N$. They are represented by Ramanujan ordered numbers [19] of type $2^{-l} + 2^{-m}$ where l and m are integers. The DCT coefficients are computed using shift and addition operations only thereby avoiding the floating-point multipliers. The proposed algorithm thus achieves a considerable reduction in computational complexity as compared with the existing multi-dimensional algorithms.

This paper is organized as follows: Section 2 gives the proposed m -dimensional polynomial transforms based Ramanujan ordered number DCT. Section 3 shows the efficacy of the proposed algorithm when compared with the

existing algorithms.

II. MULTIDIMENSIONAL DCT

The 2-D DCT described [18] is extended for the computation of the m -D DCT. The m -D DCT can be expressed as,

$$X(k_1, k_2, \dots, k_m) = \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} \dots \sum_{n_m=0}^{N_m-1} x(n_1, n_2, \dots, n_m) \cdot \cos \frac{\pi(2n_1+1)k_1}{2N_1} \dots \cos \frac{\pi(2n_m+1)k_m}{2N_m} \quad (1)$$

$k_i = 0, 1, \dots, N_i - 1; \quad i = 1, 2, \dots, m$

Let us assume that N_i ($1, 2, \dots, m$) are the numbers which are powers of 2 and that N_m is the largest number in the array. Hence we can write $N_m = 2^l$ and $N_m / N_i = 2^{l_i}$, $i = 1, 2, \dots, m-1$, $l_i \geq 0$.

If the multidimensional input array could be rearranged as

$$\begin{aligned} y(n_1, n_2, \dots, n_m) &= x(2n_1, 2n_2, \dots, 2n_m) \\ y(N_1 - 1 - n_1, n_2, \dots, n_m) &= x(2n_1 + 1, 2n_2, \dots, 2n_m) \\ &\dots\dots\dots \\ y(N_1 - 1 - n_1, N_2 - 1 - n_2, \dots, N_m - 1 - n_m) &= x(2n_1 + 1, 2n_2 + 1, \dots, 2n_m + 1), \end{aligned} \quad (2)$$

$n_i = 0, 1, \dots, N_i / 2 - 1, \quad i = 1, 2, \dots, m$

A. Algorithm for Multidimensional DCT

Let us define

$$A(k_1, k_2, \dots, k_m) = \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} \dots \sum_{n_m=0}^{N_m-1} y(n_1, n_2, \dots, n_m) \cdot \cos \frac{\pi(4n_1+1)k_1}{2N_1} \dots \cos \frac{\pi(4n_{m-2}+1)k_{m-2}}{2N_{m-2}} \cdot \cos \left[\frac{\pi(4n_{m-1}+1)k_{m-1}}{2N_{m-1}} + \dots + \frac{\pi(4n_m+1)k_m}{2N_m} \right] \quad (3)$$

and

$$B(k_1, k_2, \dots, k_m) = \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} \dots \sum_{n_m=0}^{N_m-1} y(n_1, n_2, \dots, n_m) \cdot \cos \frac{\pi(4n_1+1)k_1}{2N_1} \dots \cos \frac{\pi(4n_{m-2}+1)k_{m-2}}{2N_{m-2}} \cdot \cos \left[\frac{\pi(4n_{m-1}+1)k_{m-1}}{2N_{m-1}} - \dots - \frac{\pi(4n_m+1)k_m}{2N_m} \right] \quad (4)$$

Thus equation (5) can be computed as

$$X(k_1, k_2, \dots, k_m) = \frac{1}{2} [A(k_1, k_2, \dots, k_m) + B(k_1, k_2, \dots, k_m)] \quad (5)$$

Let $f_{m-1}(n_m)$ be the least non-negative remainder of $[(4f_{m-1}+1)n_m + f_{m-1}] \bmod N_{m-1}$. Similar to 2-D DCT, we can write

$$\begin{aligned} A(k_1, k_2, \dots, k_m) &= \sum_{n_1=0}^{N_1-1} \dots \sum_{n_{m-2}=0}^{N_{m-2}-1} \sum_{f_{m-1}=0}^{N_{m-1}-1} \sum_{n_m=0}^{N_m-1} y(n_1, \dots, n_{m-2}, f_{m-1}(n_m), n_m) \cdot \cos \frac{\pi(4n_1+1)k_1}{2N_1} \dots \cos \frac{\pi(4n_{m-2}+1)k_{m-2}}{2N_{m-2}} \cdot \cos \left[\frac{\pi(4n_{m-1}+1)(2^{l_{m-1}}(4f_{m-1}+1))k_{m-1} + k_m}{2N_m} \right] \end{aligned} \quad (6)$$

and

$$\begin{aligned} B(k_1, k_2, \dots, k_m) &= \sum_{n_1=0}^{N_1-1} \dots \sum_{n_{m-2}=0}^{N_{m-2}-1} \sum_{f_{m-1}=0}^{N_{m-1}-1} \sum_{n_m=0}^{N_m-1} y(n_1, \dots, n_{m-2}, f_{m-1}(n_m), n_m) \cdot \cos \frac{\pi(4n_1+1)k_1}{2N_1} \dots \cos \frac{\pi(4n_{m-2}+1)k_{m-2}}{2N_{m-2}} \cdot \cos \left[\frac{\pi(4n_{m-1}+1)(2^{l_{m-1}}(4f_{m-1}+1))k_{m-1} - k_m}{2N_m} \right] \end{aligned} \quad (7)$$

We could define

$$\begin{aligned} V_{k_1, k_2, \dots, k_{m-2}, f_{m-1}}(l) &= \sum_{n_1=0}^{N_1-1} \dots \sum_{n_{m-2}=0}^{N_{m-2}-1} \sum_{n_m=0}^{N_m-1} y(n_1, \dots, n_{m-2}, f_{m-1}(n_m), n_m) \cdot \cos \frac{\pi(4n_1+1)k_1}{2N_1} \dots \cos \frac{\pi(4n_{m-2}+1)k_{m-2}}{2N_{m-2}} \cdot \cos \left[\frac{\pi(4n_m+1)l}{2N_m} \right] \end{aligned} \quad (8)$$

To be the $(m-1)$ dimensional DCT with size $N_1 \times N_2 \times \dots \times N_{m-2} \times N_m$. Thus equation (3) and (4) become

$$A(k_1, k_2, \dots, k_m) = \sum_{f_{m-1}=0}^{N_{m-1}-1} V_{k_1, k_2, \dots, k_{m-2}, f_{m-1}}(2^{l_{m-1}}(4f_{m-1}+1)k_{m-1} + k_m) \quad (9)$$

$$B(k_1, k_2, \dots, k_m) \quad (8)$$

$$= \sum_{f_{m-1}=0}^{N_{m-1}-1} V_{k_1, k_2, \dots, k_{m-2}, f_{m-1}}(2^{l_{m-1}}(4f_{m-1}+1)k_{m-1} - k_m) \quad (10)$$

Let us form the polynomial as

$$\begin{aligned} B_{k_1, k_2, \dots, k_{m-1}}(z) &= \sum_{k_m=0}^{N_m-1} B(k_1, k_2, \dots, k_m) z^{k_m} - \sum_{k_m=1}^{N_m} A(k_1, k_2, \dots, 2N_m - k_m) z^{k_m} \\ &= \sum_{k_m=0}^{2N_m-1} B(k_1, k_2, \dots, k_m) z^{k_m} \end{aligned} \quad (11)$$

which can be expressed as

$$B_{k_1, k_2, \dots, k_{m-1}}(z) \equiv C_{k_1, k_2, \dots, k_{m-1}}(z) z^{2^{l_{m-1}k_{m-1}}} \bmod z^{2N_m} + 1 \quad (12)$$

where

$$C_{k_1, k_2, \dots, k_{m-1}}(z) \equiv \sum_{f_{m-1}=0}^{N_{m-1}-1} U_{k_1, k_2, \dots, k_{m-2}, f_{m-1}}(z) \hat{z}^{f_{m-1}k_{m-1}} \bmod z^{2N_m} + 1 \quad (13)$$

$$k_i = 0, 1, 2, \dots, N_i - 1, \quad i = 1, 2, \dots, m-1$$

$$\hat{z} \equiv z^{2^{l_{m-1}+2}} \bmod z^{2N_m} + 1$$

and

$$U_{k_1, k_2, \dots, k_{m-2}, f_{m-1}}(z) = \sum_{l=0}^{2N_{m-1}-1} V_{k_1, k_2, \dots, k_{m-2}, f_{m-1}}(l) z^l \quad (14)$$

which is the generating polynomial of $V_{k_1, k_2, \dots, k_{m-2}, f_{m-1}}(l)$.

The flow of the algorithm can be summarized as follows:

- 1) Compute the $N_{m-1}(m-1)$ -dimensional DCT according to equation (5). If $(m-1) > 1$, similar procedure is used to decompose the $(m-1)$ -D DCT into smaller sizes. If $(m-1) = 1$, the 1-D DCT is computed using the multiplierless Ramanujan ordered number representation as in equation (16) and equation (17) of [18].
- 2) Compute $N_1 N_2 \dots N_{m-2}$ 1-D polynomial transforms using equation (13), and then compute $B_{k_1, k_2, \dots, k_{m-1}}(z)$ using equation (12).
- 3) $A(k_1, k_2, \dots, k_m)$ and $B(k_1, k_2, \dots, k_m)$ can then be computed from the definition of $B_{k_1, k_2, \dots, k_{m-1}}(z)$ using equation (11). The Coefficients $X(k_1, k_2, \dots, k_m)$ are then computed by equation (5), which requires $N - N/N_m - N/N_{m-1} + 1$ additions where $N = N_1 N_2 \dots N_m$.

B. Computational Complexity of the Algorithm

Computational complexity of the proposed algorithm can be explained by considering all the operations required in the three steps listed above. Hence, for the input data of size $N_1 \times N_2 \times \dots \times N_m$, the computational complexity is

$$\begin{aligned} S_m(N_1, N_2, \dots, N_m) &= N_{m-1} S_{m-1}(N_1, N_2, \dots, N_{m-2}, N_m) \\ A_m(N_1, N_2, \dots, N_m) &= N_{m-1} A_{m-1}(N_1, N_2, \dots, N_{m-2}, N_m) + N \log_2 N_{m-1} \\ &\quad + N - \frac{3N}{2N_m} - \frac{N}{N_{m-1}} + \frac{N}{N_{m-1}N_m} + 1 \end{aligned} \quad (15)$$

where $N = N_1 N_2 \dots N_m$, $M_m(N_1, N_2, \dots, N_m)$ is the number

of shift operations and $A_m(N_1, N_2, \dots, N_m)$ is the number of additions.

Considering that the computational complexity of the 1-D DCT using Ramanujan Ordered number is $N/2 \log_2 N$ number of shift operations and $(3N/2 \log_2 N) - N + 1$ number of additions, the total number of shift operations and additions needed by the m-D DCT can be given as

$$\begin{aligned} S_m(N_1, N_2, \dots, N_m) &= \frac{1}{2} N \log_2 N_m \\ A_m(N_1, N_2, \dots, N_m) &= \frac{3}{2} N \log_2 N_m + N \log_2 \frac{N}{N_m} + (m-2)N \\ &\quad - \frac{3m-5}{2} \frac{N}{N_m} - N \left(1 - \frac{1}{N_m} \right) \left(\frac{1}{N_1} + \dots + \frac{1}{N_{m-1}} \right) + (m-1) \end{aligned} \quad (16)$$

Although the proposed algorithm requires same number of additions as required by the algorithm in [17], the requirement of multipliers is completely eliminated by using the Ramanujan ordered number.

TABLE I: COMPARISON OF COMPUTATIONAL COMPLEXITY OF RDCT

Operation	1-D RDCT [20]	2-D RDCT [18]	m-D RDCT proposed
Shifts	$N/2 \log_2 N$	$N^2/2 \log_2 N$	$\frac{1}{2} N \log_2 N_m$
Additions	$(3N/2 \log_2 N) - N + 1$	$\frac{5N^2}{2} \log_2 N - 3N/2 + 2$	$\frac{3}{2} N \log_2 N_m + N \log_2 \frac{N}{N_m} + (m-2)N - \frac{3m-5}{2} \frac{N}{N_m} - N \left(1 - \frac{1}{N_m} \right) \left(\frac{1}{N_1} + \dots + \frac{1}{N_{m-1}} \right) + (m-1)$

TABLE II: PERFORMANCE EVALUATION OF VARIOUS MULTIDIMENSIONAL DCT ALGORITHMS IN TERMS OF MULTIPLIER OPERATIONS

Input size	PT based m-D RDCT	Recursive RDCT [18]	Tensor product [12]	Zeng's PT [17]
	Shifts	Shifts	Floating-point multipliers	Floating-point multipliers
8×8	96	192	96	96
16×16	512	1024	512	512
32×32	2560	5120	2560	2560
4×4×4	64	128	64	64
8×8×8	768	1536	768	768

TABLE III: PERFORMANCE EVALUATION OF VARIOUS MULTIDIMENSIONAL DCT ALGORITHMS IN TERMS OF ADDITION OPERATIONS

Input size	PT based m-D RDCT	Recursive RDCT [18]	Tensor product [12]	Zeng's PT [17]
8×8	466	6561	470	466
16×16	2722	43681	2554	2722
32×32	14082	263169	12786	14082
4×4×4	448	729	446	448
8×8×8	5600	24389	5594	5600

III. SIMULATION RESULTS

The performance of the proposed algorithm can be evaluated by comparing the computational complexity of the algorithms with respect to the number of operations used to compute the DCT. Table I tabulates the computational complexity of the multiplierless Ramanujan ordered number DCT(RDCT) for 1-D RDCT, 2-D RDCT generalized with m-D RDCT in terms of shift operations and addition

operations.

Table II compares the performance of the proposed m-D RDCT with the very commonly used multi-dimensional DCT algorithms in terms of the shift/multiplication operation required. The proposed algorithm requires same number of shift operations as much as the floating-point multipliers required by the tensor product method and the Zeng's PT method. Hence the proposed algorithm is completely multiplierless. When compared with the multiplierless row-column RDCT algorithm, the proposed one reduces the complexity by 66%.

Table III gives an overview of the performance of the proposed algorithm with respect to the number of addition operation required. From the table it is clear that the proposed m-D RDCT requires no additional adders in order to compensate for the floating-point multipliers, which clearly indicates reduction in computational complexity when compared with the other multidimensional algorithms. But there is an increase in number of adders required when compared with row-column RDCT by 0.1% which is very marginal. Thus, the main advantage of the proposed algorithm is that it maintains the simplicity of the polynomial transform structure by utilizing the symmetric properties.

IV. CONCLUSIONS

Based on the method of obtaining the multiplierless structure for computing 1-D DCT using Ramanujan ordered Number, the proposed algorithm presents a new method which generalizes to compute multidimensional DCT utilizing the benefits of polynomial transform. The use of Ramanujan ordered number completely eliminates the floating-point multiplications by shift operations only. The Polynomial transform further reduces the complexity of the algorithm to compute the multidimensional DCT. Symmetric properties of the transform are utilized to reduce the number of arithmetic operations compared with the row-column method. The requirement of the M being a Ramanujan ordered number is usually satisfied as the multidimensional DCT operates on a block length of either 4 or 8 or 16. Thus the proposed fast algorithm can be exploited to reduce the overall computational and structural complexity.

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