Reduced Complexity QRM-MLD for MIMO System

Jinyong Lee, Youngseh Kim, Kanghoon Kim, and Younglok Kim

Abstract—This paper proposes reduced complexity QR decomposition with M-algorithm (QRM) maximum likelihood detection (MLD) by removing superfluous paths according to the maximum squared Euclidean distance. As a result of computer simulation, the proposed method can have 56% reduction of computational complexity compared with the conventional QRM-MLD without any performance degradation at 20dB SNR. Compare with MMSE method, the 3.0% BER of conventional method is reduced to 0.04% by proposed method.

Index Terms—Channel equalization, MIMO OFDM, MMSE, QRM-MLD.

I. INTRODUCTION

In the next generation wireless communication system, especially 3rd generation partnership project (3GPP) long term evolution (LTE), multiple-input multiple-output (MIMO) technique is adopted in order to achieve high data throughput. Spatial multiplexing, which is one of MIMO techniques, required high performance data detection algorithm with low computational complexity. In general, maximum likelihood detection (MLD) approach has the highest performance but its computational complexity is exponentially increased by the number of transmitter antennas and the modulation level in data modem scheme [1]. Therefore QRM-MLD is used for data detection for low computational complexity [2]. However, computational complexity is not low enough in case of using QRM-MLD.

In this paper, we propose reduced complexity of the QRM-MLD which can decrease computational complexity by removing superfluous paths according to the maximum squared Euclidean distance. And we performed its bit error ratio (BER) performance by computer simulations, and compared computational complexity with minimum mean square error (MMSE) and conventional QRM-MLD methods by using real operations (ROPs).

This paper is organized as follows: The system modeling is introduced and previous detection methods are reviewed in section II. In section III, the reduced complexity QRM-MLD method is proposed. The BER performance and comparison complexity are evaluated by simulations in section IV, and the conclusion is remarked in Section V.

II. SIGNAL DETECTION METHODS

A. System Modeling

Let $n_t$ is transmit antenna and $n_r$ is receive antenna in the spatially multiplexed MIMO system. Then the received signal vector can be denoted by

$$\mathbf{y} = \mathbf{H} \mathbf{x} + \mathbf{n}$$  \hspace{1cm} (1)

where $\mathbf{x} = [x_1, x_2, \cdots, x_{n_t}]^T$ is the $n_t$ dimensional transmit vector, and $\mathbf{y}$ is the $n_r$ dimensional received vector, $\mathbf{n}$ is the additive white Gaussian noise (AWGN) vector. Channel matrix $\mathbf{H}$ is an $n_t \times n_r$ dimension with the element $h_{i,j}$ being the channel gain from $i$-th transmit antenna to the $j$-th receive antenna.

$$\mathbf{H} = \begin{bmatrix} h_{1,1} & \cdots & h_{1,n_r} \\ \vdots & \ddots & \vdots \\ h_{n_t,1} & \cdots & h_{n_t,n_r} \end{bmatrix} = \begin{bmatrix} h_1 & h_2 & \cdots & h_{n_r} \end{bmatrix}$$  \hspace{1cm} (2)

We assume that $\mathbf{H}$ has independently and identically distributed.

B. Minimum Mean Square Error (MMSE)

MMSE is a signal detection method considering a variance of noise which is added in receiver. It uses the weight vector $\mathbf{W}$ that minimizes the MSE value between transmitted signal $\mathbf{x}$ and estimated signal $\hat{\mathbf{x}}$. MSE value can be calculated as followings.

$$E^2 = E[(\mathbf{x} - \hat{\mathbf{x}})^H(\mathbf{x} - \hat{\mathbf{x}})]$$

$$= E[(\mathbf{x} - \mathbf{W}\mathbf{y})^H(\mathbf{x} - \mathbf{W}\mathbf{y})]$$  \hspace{1cm} (3)

We can obtain the optimized solution of equation (3), by this equation

$$\mathbf{W}_{\text{MMSE}} = \left(\mathbf{H}^H \mathbf{H} + \sigma_n^2 \mathbf{I}_{n_t}\right)^{-1} \mathbf{H}^H$$  \hspace{1cm} (4)

MMSE provides the performance degradation compared with MLD, because the weight vector $\mathbf{W}$ cannot eliminate interference signal perfectly. However it has much less computational complexity.

C. Maximum Likelihood Detection (MLD)

MLD is known as the optimal signal detection method for spatially multiplexed MIMO systems. MLD selects the transmitted symbol which has the smallest Euclidean distance with constellation points that transmitted symbol can be located. The decision value for estimated signal $\hat{\mathbf{x}}$ is represented as followings.

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x} \in \mathcal{C}} \| \mathbf{y} - \mathbf{Hx} \|^2$$  \hspace{1cm} (5)
The number of \( C^{n_t} \) computational complexity is required for MLD, where \( C \) is the number of constellation points.

\[
\begin{align*}
\text{Fig. 1. Equivalent signal model using QR decomposition} \\
\text{Fig. 2. Tree search based structure for QRM-MLD} \\
\end{align*}
\]

\[
\text{Fig. 3. Example of reduced computational QRM-MLD}
\]

\[ y \approx Q^H y = Q^H Q R x + Q^H n = R x + n \]  \hspace{1cm} (6)

\[ \tilde{y}_1 = L_{1,1} x_1 + L_{1,2} x_2 + L_{1,3} x_3 + L_{1,4} x_4 + \eta_1 \\
\tilde{y}_2 = L_{2,2} x_1 + L_{2,3} x_3 + L_{2,4} x_4 + \eta_2 \\
\tilde{y}_3 = L_{3,3} x_3 + L_{3,4} x_4 + \eta_3 \\
\tilde{y}_4 = L_{4,4} x_4 + \eta_4 \]  \hspace{1cm} (7)

The equation for MLD decision value is the same as follows,

\[
\hat{x} = \arg \min_{\{ \tilde{x}_i \}} \left\{ \sum_{i=1}^{M} \left| \tilde{y}_i - \sum_{j=1}^{N_t} R_{i,j} \tilde{x}_j \right|^2 \right\} \]  \hspace{1cm} (8)

where \( \hat{x}_j \) is one of \( M \) constellation points. Using QRM-MLD, computational complexity is reduced to \( (C + M \cdot C + M \cdot C + M) \), when both \( n_t \) and \( n_r \) are four.

\[ \| y - H \hat{x}_{\text{MMSE}} \|^2 \geq \| y - H \hat{x}_{\text{QRM-MLD}} \|^2 \]  \hspace{1cm} (9)

where \( \hat{x} \) is selected from the constellation of each symbol. Euclidean distance of MMSE is always bigger than that of QRM-MLD, so it is set up for the maximum limit. The paths which have more accumulated matrix value than the maximum limit are eliminated. Using the proposed method, we can reduce computational complexity.

The proposed algorithm is summarized as following steps:

1. Step1: Estimate the transmit vector by multiplying \( W_{\text{MMSE}} \) to the received vector.

\[
\hat{x}_{\text{MMSE}} = Q(W_{\text{MMSE}} \cdot y) \]  \hspace{1cm} (10)

2. Step2: Calculate Euclidean distance of estimated vector using MMSE in order to decide the maximum distance.

\[
\text{MaxDist} = \| y - R \hat{x}_{\text{MMSE}} \| 
\]  \hspace{1cm} (11)

3. Step3: In every stage, we select \( M \) paths that make the smallest accumulated squared Euclidean distances in order to avoid redundant computation. Fig. 3 explains proposed scheme.

\[ \text{TABLE I: SIMULATION PARAMETERS} \]

<table>
<thead>
<tr>
<th>Parameters</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmission BW</td>
<td>10 MHz</td>
</tr>
<tr>
<td>Sampling frequency</td>
<td>15.36 MHz</td>
</tr>
<tr>
<td>FFT size</td>
<td>1,024</td>
</tr>
<tr>
<td>Subcarrier spacing</td>
<td>15kHz</td>
</tr>
<tr>
<td>Modulation</td>
<td>16 QAM</td>
</tr>
<tr>
<td>Number of Tx antennas</td>
<td>4</td>
</tr>
<tr>
<td>Number of Rx antennas</td>
<td>4</td>
</tr>
<tr>
<td>Channel environment</td>
<td>Typical Urban 6</td>
</tr>
<tr>
<td>Channel estimation</td>
<td>Perfect</td>
</tr>
</tbody>
</table>

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B. Performance and Computational Complexity Analysis

In this section, we have compared the performance and computational complexity of the proposed scheme and conventional QRM-MLD. Fig. 4 shows the BER in dB scale. In simulation results, we can see that there is no performance degradation at all SNR compared with the conventional QRM-MLD.

![Fig. 4. BER performance of reduced complexity QRM-MLD](image)

In order to calculate computational complexity, we define computational complexity of complex operation as shown in Table II.

In Table III, we compare the computational complexity of the signal detection scheme from the view point of the Table II. It is shown that reduced complexity QRM-MLD which reduces 59 % computational complexity at 20dB SNR compared with the conventional QRM-MLD.

<table>
<thead>
<tr>
<th>Complex operation</th>
<th>Number of real operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiplication</td>
<td>4 ROP</td>
</tr>
<tr>
<td>Division</td>
<td>10 ROP</td>
</tr>
</tbody>
</table>

V. CONCLUSION

In this paper, we propose reduced complexity QRM-MLD which can eliminate unnecessary paths using maximum squared Euclidean distance. Simulation results have shown that the proposed scheme can achieve the same performance as conventional QRM-MLD but with smaller computational complexity.

<table>
<thead>
<tr>
<th>SNR (Eb/No) Type</th>
<th>15 dB</th>
<th>20 dB</th>
<th>Performance degradation at 0.1 % BER</th>
</tr>
</thead>
<tbody>
<tr>
<td>QRM-MLD (M=16)</td>
<td>100 %</td>
<td>100 %</td>
<td>-</td>
</tr>
</tbody>
</table>

REFERENCES


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