Control of the Magnetic Suspension System with a Three-degree-of-freedom Using RBF Neural Network Controller

Mohammad Saberi, Hamid Altafi, and Seyyed Morteza Alizadeh

***Abstract*—In this paper an intelligent method is proposed for controlling a kind of magnetic suspension system with 3 degree of freedom. At first, the dynamic of the magnetic suspension system and the related equations are presented. Regarding unstable nature and non-linearity of magnetic suspension system using techniques of liner control for achieving optimal performance so that all requirements of system are met in all domains is difficult. Then optimal controlling input for magnetic suspension system is designed using optimal control method, linear quadratic regulator (LQR) and required computations. For designing the neural network controller, Radial Basis Function (RBF), we use the results gained by LQR controller. The simulation results are performed using MATLAB software and performance of proposed controlling method was approved.**

***Index Terms*—Magnetic suspension, Linear quadratic regulator, Radial basis function, Neural controller.**

I. Introduction

Magnetic suspension systems is becoming increasingly popular for high–speed ground transportation systems, magnetic bearings, vibration isolation systems and fast-tool servo systems[1]-[4]. Due to the features of the open-loop instability and inherent nonlinearities in electromechanical dynamics of the magnetic suspension systems, the development of a high-performance control design for the position control of the levitated object is very important. In general, the electromechanical dynamics of the magnetic suspension systems are represented by a nonlinear model, which consists of the state variables of magnet deviation, vertical velocity, dynamic magnet force. Therefore, applications of the feedback linearization control techniques have been presented in many studies for the magnetic suspension systems [5]–[8]. In [5] and [6], feedback linearization techniques were adopted to transform the system model into an equivalent model with a simple form. Although the dynamic model of the magnetic suspension system was simplified, only the nominal system parameters were considered in the feedback linearization design. This usually leads to problems with deteriorated performance and instability since the system parameters are usually varied with thermal drift. Moreover, in [7], the nonlinear dynamics were approximated by the use of Taylor's series expansion. However, the high-order terms in Taylor's series expansion were neglected for simplifying the original dynamic model to a second-order differential equation. As a result, the development of a high-performance controller based on this simplified model was very difficult to complete. Furthermore, a robust feedback linearization controller for an electromagnetic suspension system was presented in [8]. Although the stability of the control system was guaranteed theoretically, it seems that the relatively large overshoots and oscillation in transients existed in their experimental results. This deteriorated the transient performance and resulted in impractical applications of the suspension systems. Therefore, some adaptation laws should be applied to solve the mentioned difficulty. On the other hand, an adaptive robust nonlinear controller is proposed in [9] for a magnetic suspension system via the back stepping design approach. First, a proportional–integral controller is designed to stabilize the position error of the levitated object. Then, an adaptive robust nonlinear controller is designed to attenuate the effects of parameter uncertainties.

The major contributions of this study are: 1) the successful development of an compound controlling method in which RBF neural network is used in order to emulate behavior of the LQR optimal controller. 2) the Successful application of proposed controlling method for controlling magnetic suspension system with 3 degree of freedom

In this work, application of RBF neural network in controlling a magnetic suspension system with 3 degree of freedom is considered. At first, dynamics of this system and related equations are presented. Then system becomes linear around the set point using linearization control techniques. Optimal controlling input for magnetic suspension system is designed using LQR optimal control method and required computations.

Intelligent control approaches such as neural network and fuzzy system do not require mathematical models and have the ability to approximate nonlinear systems. Therefore, there were many researchers who use intelligent control approaches to represent complex plants and construct advanced controllers [10]. Moreover, the locally tuned and overlapped receptive field is a well-known structure that has been studied in the regions of cerebral cortex, visual cortex, and so on [10]. Based on the biological receptive fields, the radial basis function network (RBFN) that employs local receptive fields to perform function mappings was proposed in [11]. Furthermore, the RBFN has a faster convergence property than a multilayer perceptron (MLP) since only the connective weights between the hidden layer and the output layer of the network are adjusted during training to reduce the

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computational requirements.

In designing neural network controller for magnetic suspension system, in the first stage (training), we use the results gained by the LQR optimal controller to model the system. In the second stage (testing), model obtained from magnetic suspension system is used for designing controller based on RBF neural network; Finally RBF neural controller is designed.

It is concluded that the proposed RBF neural network controller can replace the LQR optimal controller.

II. CONTROL SYSTEM MODELING

The comprehensive explanations about mechanical parts of the suspension magnetic system have been explained in [12]. with the assumption of a single levitated mass as shown in Fig. 1, the LQR controller are designed for both suspension and guidance. There m is the levitated mass (including suspension frame and magnet), f is the dynamic magnet force, zT is the track disturbance and zt is the magnet deviation. S0 is the nominal magnet gap, s is the small deviation from the nominal gap, L0 = Le + Ls is the whole inductance, with the effective inductance Le and the constant stray inductance Ls. R is the resistance, I0 is the nominal current and u is the small deviation from the nominal magnet voltage R I0.

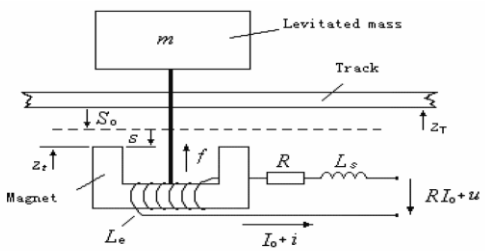

Fig. 1. Model of a single mass vehicle for vertical motion

The linear equation of the single mass magnetic suspension system can be derived with linearized equations at the working point as shown in [13]–[16]

$$\ddot{s} = -\frac{(1-\eta)}{m} l_0 \frac{I_0}{S_0} i + \frac{(1-\eta)}{m} l_0 \left(\frac{I_0}{S_0}\right)^2 s = -\frac{P_i}{m} i + \frac{P_s}{m} s \tag{1}$$

$$\dot{i} = -\frac{R}{L_o} i + (1-\eta)\frac{I_o}{S_o}\dot{s} + \frac{1}{L_o} u \tag{2}$$

where

$$\eta = \frac{L_s}{L_o}\ ,\ P_i = \frac{(1-\eta)L_o I_o}{S_o}\ ,\ P_s = \frac{(1-\eta)L_o I_o^2}{s_o^2}$$

Considering the dynamic magnet force $P_i i - P_s s$,then is

$$\dot{f} = P_i \dot{i} - P_s \dot{s} \tag{3}$$

Substituting (2) into (3) and taking note of

$i = \frac{P_s}{P_i} s + \frac{1}{P_i} f$, there is

$$\dot{f} = -\frac{P_s}{T} s - P_s \eta \dot{s} + \frac{(1-\eta)P_s}{P_i} u \tag{4}$$

where $T = \frac{L_0}{R}$ is the time constant. In order to accord with the vehicle coordinate system the transformation is made as below

$$Z_t = -S + Z_T\ ,\ \dot{Z}_t = -\dot{Z} + \dot{Z}_T$$

Selecting magnet deviation z_t , vertical velocity $\dot{z}_t$ and dynamic magnet force f as the state variables, that is $x(t) = \begin{bmatrix} z_t & \dot{z}_t & f \end{bmatrix}^T$, the following state equation is obtained

$$\begin{bmatrix} \dot{Z}_t \\ \ddot{Z}_t \\ \dot{f} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & \frac{1}{m} \\ \frac{P_s}{T} & P_s\eta & -\frac{1}{T} \end{bmatrix} \begin{bmatrix} Z_t \\ \dot{Z}_t \\ f \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{(1-\eta)P_s}{P_i} \end{bmatrix} u + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -P_s/T & P_s\eta \end{bmatrix} \begin{bmatrix} Z_T \\ \dot{Z}_T \end{bmatrix}$$

i.e.

$$\dot{x}(t) = Ax(t) + B\,u(t) + E\,V(t) \tag{5}$$

With magnet gap deviation z_t , vertical velocity $\dot{z}_t$ and vertical acceleration $\ddot{z}_t$ as the measurements the following output equation is obtained (in practice $\dot{z}_t$ is computed by an observer [17], [18])

$$\begin{bmatrix} Z_t \\ \dot{Z}_t \\ \ddot{Z}_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{m} \end{bmatrix} \begin{bmatrix} Z_t \\ \dot{Z}_t \\ f \end{bmatrix}$$

i.e. $$y(t) = c\,x(t) \tag{6}$$

III. LQR OPTIMAL CONTROL STRATEGY

The LQR control scheme optimizes a performance index J of the form

$$j = \frac{1}{2}\int_0^\infty \left[x^T(t) c^T Q\ C\ x(t) + u^T(t) R\ U(t)\right] \tag{7}$$

where Q and R are positive semi-definite and positive definite weighting matrices, respectively. The control input vector u is defined as

$$u(t) = -kx(t), \tag{8}$$

where K is a constant gain matrix found as

$$k = R^{-1} B^T P \tag{9}$$

And P is a unique positive semi-definite solution matrix to the following algebraic Riccati equation (ARE):

$$PA + A^T P - PBR^{-1}B^T P + C^T QC = 0 \tag{10}$$

The equations mentioned above shows that the key to design optimal controller is to choose the appropriate weighting matrices Q and R, and according to this, to calculate the P in Riccati matrix algebraic equation, then the feedback gain K can be solved.

The weightings used for the control system design for minimum control energy are

$$Q = diag\begin{bmatrix}100 & 10 & 10\end{bmatrix}\ ;\ R = 1$$

By performing required computations and appropriate selection of parameters in designing the LQR optimal controller, the state variables and as a result, the optimal controlled input gained by LQR becomes convergent to zero. This convergence to zero is one of the characteristics of the LQR optimal controller.

IV. Structure of Neural Network

A. RBF Network Structure

Recently, the RBF neural network has become popular because of its structural simplicity and training efficiency [19], [20]. An RBF neural network consists of two fully connected layers, namely hidden and output layers as shown in Fig. 2. The input nodes are directly connected to the hidden layer neurons. The output of the jth hidden neuron can be written as

$$h_j = \frac{\varphi\left|X - c_j\right|}{\sigma_j} \tag{11}$$

where h_j is the output of the jth neuron, φ, the nonlinear radial basis transfer function, X , the input vector, c_j ,neuron's center and σ_j, the center spread parameter. Spread denotes distribution of the radial radix function and its value shows the ratio range of radial radix neuron to inputs response.

The nonlinearity of the RBF neural network is due to its transfer function φ. The most commonly used type of radial basis function is the Gaussian given as

$$h_j = \exp\left[-\frac{\left|X - c_j\right|^2}{\sigma_j^{\ 2}}\right] \tag{12}$$

The neurons of the output layer have a linear transfer function. It is simply the weighted summation of outputs of all hidden neurons connected to that output neuron. For the kth neuron, the output Y_k is

$$Y_k = \sum_{j=1}^{m} W_{kj} h_j \tag{13}$$

where W_{kj} is the synaptic weight connecting the hidden neuron j to the output neuron k and m is the number of hidden layer neurons.

B. Learning Algorithm

There are various radial radix neuron function. In this paper,

it is:

$$radbas\,(n) = e^{-n^2} \tag{14}$$

The network input is P. The initial power value for the first layer is P′. The neuron amount is equal to input sample vector amount. The output a1 and T for the first layer are known and the threshold b1 for the first layer is equal to value spread. For the first layer, the power-value and threshold of neural network bring the minimum aim function value, i.e.

$$J = \frac{1}{2}\sum_{p=1}^{L}\left\|d_p - y_p\right\|^2 = \frac{1}{2}\sum_{p=1}^{L}\sum_{p=1}^{L}(d_{kp} - y_{kp})^2 \tag{15}$$

The power-value and threshold for the second layer can be gained by means of equation (15) and (16).

$$LW_{2,1}a_1 + b_2 = T \tag{16}$$

Every iteration computation of neural networks will reduce the output error and increase a neuron. If the output error is smaller than the set value, the iteration process will be ended. Otherwise, the process will be continued until the error is small enough or the number of neurons equals to the set value.

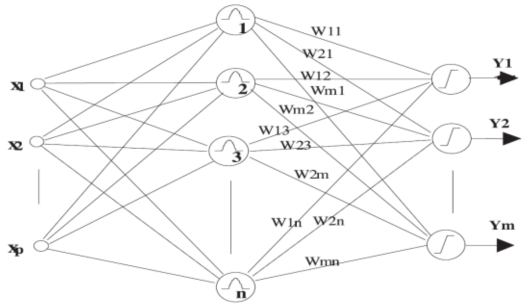

Fig. 2. The structure of RBF neural network.

V. RBF Neural Controller

After designing the LQR optimal controller for magnetic suspension system with 3 degree of freedom, it is turn to designing RBF neural controller.

For training the RBF neural network, we use the results gained by the LQR optimal controller to model the system. The block diagram of this process is shown in Fig. 3. Thus optimal applied input for magnetic suspension system is obtained by applying results of the LQR optimal controller and appropriate selection of parameters of RBF.

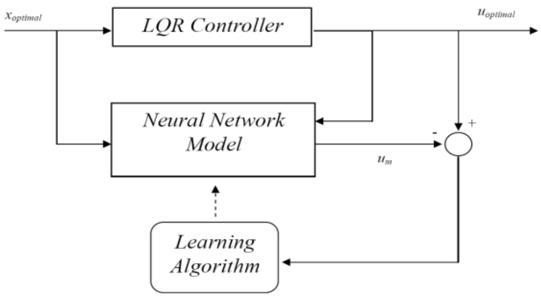

Fig. 3. Block diagram of modeling system when learning neural system

It should be noted that in RBF, $X_{Optimal}$ (desired state variables for magnetic suspension system) and $U_{Optimal}$ (optimal input for system) are considered as inputs and optimal network is obtained.

Having completed learning RBF, the controller based on RBF network is designed.

Application of neural controller while testing RBF leads to convergence of all state variables of magnetic suspension system to zero. As testing RBF neural network, using neural controller causes that all the state variables of the suspension magnet system converge on zero like LQR optimal method.

VI. Editorial Policy

A. Parameters

Parameters of simulation used in this paper are listed in Tables I and II.

TABLE I: Parameters of RBF Neural Networ

Parameter	Value
SPREAD	4
Goal	10^{-2}

TABLE II: Parameters of Magnetic Suspension System

parameter	Value
P_s	0.5 HA^2/m^2
η	0.5
S_0	1m
I_0	1A
L_0	1H
P_i	0.5 HA/m
m	0.2 Kg
T	1 Sec

According to the theoretical calculations which are mentioned in this paper, we can say that converging state variables on zero is one of the characteristics of the LQR optimal controller.

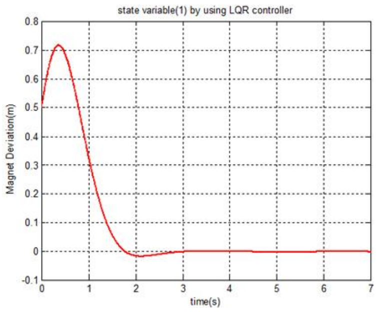

Fig. 4. First state variable by using LQR controller

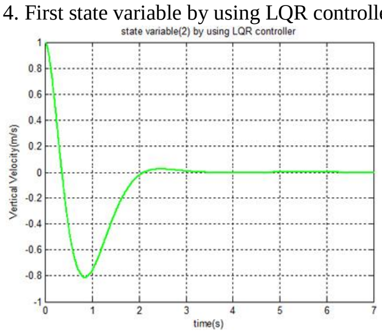

Fig. 5. Second state variable by using LQR controller

The optimal applied input for the system will be gained after considering the theoretical calculations and choosing adequate parameters in LQR optimal controller designing method. According to the equation of the LQR optimal controller, optimal applied input for the system should be converge on zero as the state variables of the system converge on zero, which can be observed in Fig. 7.

Optimal applied input for the system during learning RBF neural network is shown in Fig. 8. It should be noted that the number of neurons in input, hidden and output layers of RBF neural network are 4, 4 and 1, respectively. The number of learning steps of the network is 4.

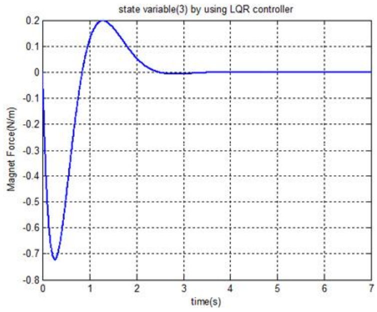

Fig. 6. Third state variable by using LQR controller

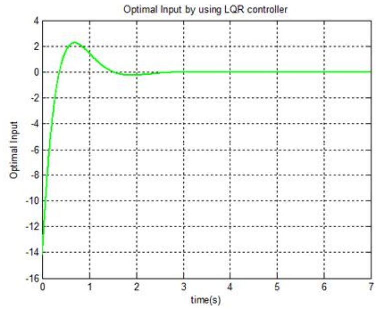

Fig. 7. Optimal input applied on system using LQR method

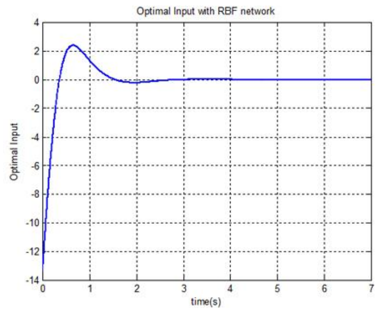

Fig. 8. Optimal input applied on system while training RBF network

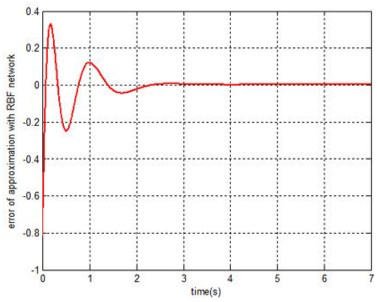

Fig. 9. Estimation error using RBF neural network

According to Fig.9, we can see that estimation error between optimal input due to LQR and input gained by RBF neural controller during learning network gradually converge on zero, which is the aim of designing.

During testing RBF neural network, using neural controller causes that all the state variables of magnetic suspension system converge on zero similar to what is done in LQR optimal control method (Figs.10 to 12).

The data used for testing network is the initial condition of the magnetic suspension system.

By designing neural controller, controlling input applied on magnetic suspension system is obtained like LQR optimal control method (fig. 13).

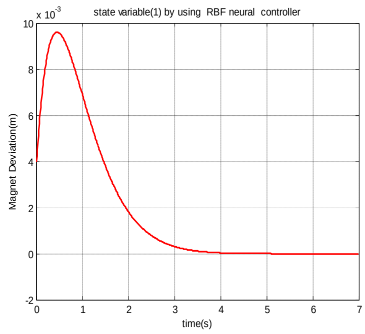

Fig. 10. First state variable using RBF neural controller

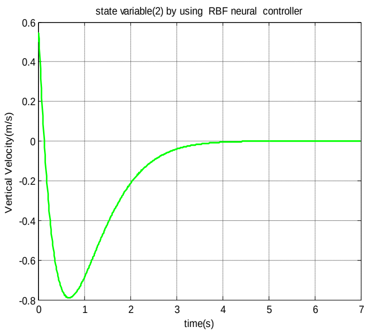

Fig. 11. Second state variable using RBF neural controller

Finally, according to figs. 14 to 16 and regarding the unique design of RBF neural controller, the outputs of the magnetic suspension system (magnetic deviation, vertical speed and vertical acceleration) with 3 degree of freedom are obtained.

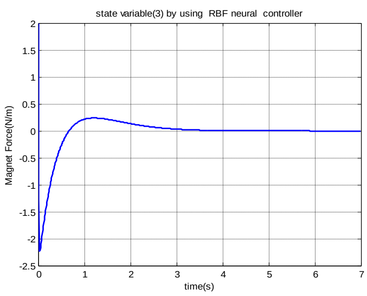

Fig. 12. Third state variable using RBF neural controller

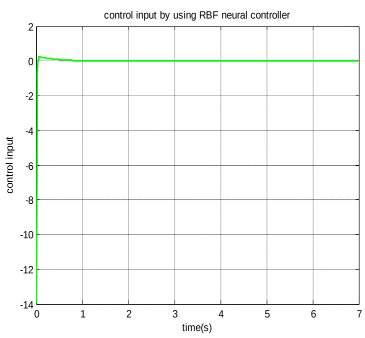

Fig. 13. Optimal controlling input using RBF neural controller

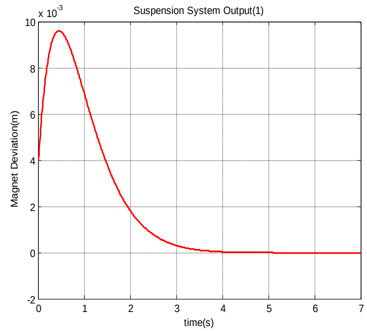

Fig. 14. First output of the system using RBF neural controller

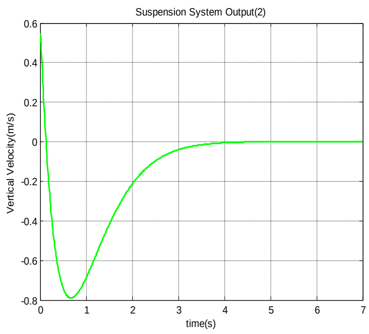

Fig. 15. Second output of the system using RBF neural controller

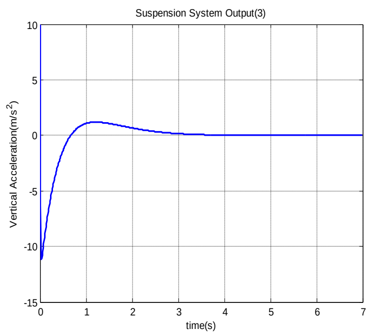

Fig. 16. Third output of the system using RBF neural controller

VII. Conclusion

In this paper development and application of RBF neural network in controlling a kind of magnetic suspension system with 3 degree of freedom were considered. First, dynamic equations of the magnetic suspension system were represented. Mathematical model of system and its related equations indicate unstable nature and extreme non linearity of magnetic suspension systems. Due these two characteristics using controlling feedback such as LQR optimal controller in them is inevitable. The system became linear around the set point using linearization control techniques. In the stage of learning neural network, results of the LQR optimal controller were used in order to providing required data for learning network. Then model obtained from training stage was used in testing stage in order to design controller based on RBF neural network controller. Finally optimal input and outputs of the magnetic suspension system were obtained.

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