Solution of Economic Load Dispatch Problem with Smooth and Non-Smooth Fuel Cost Functions Including Line Losses Using Genetic Algorithm

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***Abstract*—The paper presents an application of Genetic Algorithm (GA) to solve Economic Load Dispatch (ELD) problems with smooth and non-smooth fuel cost objective functions. Main objective of ELD is to determine the most economic generating dispatch required to satisfy the predicted load demands including line losses over a certain period of time while relaxing various equality and inequality constraints. The unit Min/Max operational constraints, effects of valve-point loading ripples and line losses are considered for the practical applications. Several cases were tested and verified, among of them, two cases of 6-units and 40-units systems including losses with smooth and non-smooth cost functions were tested, verified and compared with previous reported researches. Finally, it can be concluded that GA proves an excellent viability to optimize and solve problems of ELD. Numerical simulations indicate an improvement in total fuel cost savings.**

***Index Terms*—Economic load dispatch problem, Genetic Algorithm (GA), Smooth and non-smooth cost functions.**

I. Introduction

ELD is an optimization problem and may be solved by known means of numerical optimization. ELD is the short-term determination of the optimal output of a number of electricity generation facilities, to meet the system demand, at the lowest possible fuel cost, while serving power to the public in a robust and reliable manner [1-3]. Performing an ELD more frequently (e.g., 5 or 15 minutes rather than each hour) affects the level of costs.

Many recent works have been around Artificial Intelligence (AI) methods, on par with the development of AI optimization theories, such as Artificial Neural Networks (ANN), Simulated Annealing (SA), Genetic Algorithms (GA), Differential Evolution (DE), Particle Swarm Optimization (PSO), Evolutionary Programming (EP), and hybrid methods [3-6]. ELD algorithms for thermal unit system involving combined cycle units presented in [7]. Online solving of economic dispatch problem using neural network approach and comparing it with classical methods were presented in [8]. The Evolutionary Algorithms (EAs) are different from the conventional optimization methods, and they do not need to differentiate cost function and constraints. Theoretically, like SA, EAs converge to the global optimum solution. EAs, including Evolutionary Programming (EP), Evolutionary Strategy (ES), and GA are AI methods for optimization based on the mechanics of natural selection, such as mutation, recombination, reproduction, crossover, selection, etc [9-11]. Many researchers exerted lot of work to improve many optimization and intelligent techniques to solve ELD problems such as PSO [12-16], GA [16-17, 23], Hopfield solution [19] and SA [18].

In this paper, GA is proposed as a methodology for ELD with convex and non-convex cost functions with taking effects of valve-point loading into consideration. The data of 6 generating units and 40 generating units have taken to which are numerical tested and compared. The results are compared with [20-22].

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II. ELD Problem Formulation

The primary concern of an ELD problem is the minimization of its objective function. The total cost generated that meets the demand and satisfies all other constraints associated is selected as the objective function [1, 3]. In general, the ELD problem can be formulated mathematically as a constrained optimization problem with an objective function of the form, as illustrated in (1):

Objective Function:

$$\text{Minimize: } FC_T = \sum_{i=1}^{N} FC_i(P_i) \quad (1)$$

where FC_T is the total generation cost; N is the total number of generating units; FC_i is the power generation cost function of the i^{th} unit.

A. Classical Smooth Fuel Cost Functions

Generally, the fuel cost of a thermal generation unit is considered as a second order polynomial function (Neglecting valve-point effects) and this is called classical and smooth cost function (refer to (2)).

$$FC_i(P_i) = a_i P_i^2 + b_i P_i + c_i \quad (2)$$

where P_i is the power of the i th generating unit; a_i, b_i, c_i are the fuel cost coefficients of the i^{th} generating unit.

B. Non-Smooth Fuel Cost Functions including Valve-point Loading Effects

The generating units with multi-valve steam turbines exhibit a greater variation in the fuel cost functions. Since the valve point results in the ripples [20-21], a cost function contains higher order nonlinearity. Therefore, the cost function should be modified to consider the valve-point effects. Typically, the valve point results in, as each steam valve starts to open, the ripples like in to take account for the valve-point effects, sinusoidal functions are added to the quadratic cost functions as follows in (3) [22]:

$$FC_i(P_i) = a_iP_i^2 + b_iP_i + c_i + \left|e_i \times \sin\left(\left(f_i \times \left(P_{i,min} - P_i\right)\right)\right)\right| \quad (3)$$

where a_i, b_i, c_i are the fuel cost coefficients of the i^{th} generating unit and e_i, f_i are the coefficients of generator (i) reflecting valve-point loading effects.

These classical and non-classical models either with smooth or non-smoothed fuel cost functions are subjected to the following equality and inequality constraints:

C. Equality constraint (Power balance constraint)

The total power generated must supply the total load demand and the transmission losses, as illustrated in (4).

$$\sum_{i=1}^{N} P_i = P_{Demand} + P_{Loss} \quad (4)$$

where P_{Demand} is the total system load demand and P_{Loss} is the total line losses. Total line losses can be calculated using (5).

$$P_{Loss} = \sum_{i=1}^{N}\sum_{j=1}^{N} P_i^T B_{ij} P_j + \sum_{i=1}^{N} P_i B_{oi} + B_{oo} \quad (5)$$

B_{ij} , B_{oi} and B_{oo} are transmission line loss coefficients (P_i^T is vector transpose of all generation plants net MW, B_{ij} is square matrix of same dimension as P_i and B_{oi} is vector of same length as P_i and B_{00} is constant).

D. Inequality constraints (Maximum and minimum limits of power generation)

Each generator is constrained between its minimum and maximum limits, as shown in (6);

$$P_{i,min} \le P_i \le P_{i,max} \text{ for i=1, 2, …., N} \quad (6)$$

Where P_i is the output power of generator i; $P_{i,,min}$ and $P_{i,,max}$ are the min/max power outputs of generator i.

III. Overview of GA

Gas are well-known stochastic methods of global optimization based on the evolution theory of Darwin [4-6]. The GA is a method for solving both constrained and unconstrained optimization problems that is based on natural selection, the process that drives biological evolution. The GA uses three main types of rules at each step to create the next generation from the current population:

- Selection rules select the individuals, called parents that contribute to the population at the next generation.
- Crossover rules combine two parents to form children for the next generation.
- Mutation rules apply random changes to individual parents to form children.

Crossover rate - this rate generally should be high, about 80~95%. (However some results show that for some problems crossover rate about 60% is the best).

Mutation rate - on the other side, mutation rate should be very low.

Population size - it may be surprising that a very big population size usually does not improve performance of GA (in meaning of speed of finding solution). A good population size is about 20–30, however sometimes sizes 50–100 are reported as best. Some research also shows that best population size depends on encoding, on size of encoded string. It means, if you have a chromosome with 32 bits, the population should be say 32, but surely two times more than the best population size for chromosome with 16 bits.

The advantages of GA can be summarized as:

1. Optimizes with continuous or discrete variable.
2. Deals with a large number of variables.
3. Provides a list of optimum variables, not just a single solution.
4. Optimizes variables with extremely complex cost surfaces.
5. May encode the variables so that the optimization is done with the encoded variables and
6. Works with numerically generated data, experimental data, or analytical functions.

A. GA Solution Procedure

The GA repeatedly modifies a population of individual solutions. Some of the commonly used terminologies in GA are fitness function- which we want to minimize and population- an array of individuals. A generalized procedure for GA is summarized below.

1. The first step is to define the objective function and variables.
2. In the second step the GA parameters are selected and the population is initialized *randomly*.
3. After initialization the elite randomly and parents are selected based on the fitness value.
4. Once the parents are selected either mutation or crossover is performed form offspring.
5. The offspring is then inserted into the new population.
6. The last step is to check if the optimum solution has been achieved.

B. Terminating the Run of GA Script

This generational process is repeated until a termination condition has been reached. Common terminating conditions are: set number of iterations, set time reached, a cost that is lower than an acceptable minimum, set number of cost function evaluations, a best solution has not changed after a set number of iterations, or operator termination.

IV. GA and ELD

To use GA programming to solve ELD, the following parameters were needed for data input.

1. Number of chromosomes (that comprise a generation).
2. Number of generations.
3. Initial crossover probability (Typically 0.8).
4. Initial mutation probability (0.05 %).
5. Minimal and Maximal power outputs of each unit.
6. B-matrix of line losses.
7. Coefficients of unit fuel cost function including coefficients of valve-point loading.
8. Total load demand.

The objective function and equality, inequality constraints were written in MatLab m-files. GA parameters like population size, initial range, bounds upper and lower limits, selection criteria, crossover function, mutation function, stopping criteria and output function were set before running the program. The ranges for the variables were set in the GA and Optimization toolbox of MATLAB version 2010a [23]. The evaluation/fitness function is adopted as illustrated in (7) for evaluating the fitness of each individual in the population.

$$\text{Minimize: } FC_T = \sum_{i=1}^{N} FC_i(P_i) \qquad (7)$$

V. Case Studies and Numerical Simulations

To verify the feasibility and performance efficiency of applying GA to solve ELD with taking the effect of valve ripples into consideration, several cases were tested and investigated. Among of these, two cases will be presented. The GA based algorithm is applied to solve the six-units with line losses and neglecting effects of valve-loading as Case I. In Case II, the algorithm was applied to forty-unit system with considering valve-point loading effects and neglecting line losses. Simulations were carried out using GA and Optimization toolbox of MatLab®7.10 release 2010a version.

A. Case I: 6-Generators System with line losses (Not include Valve-loading Effects)

The Fuel cost characteristics in $/ h of the six units and the unit operating min/max (in MW) ranges [19] are given as follows:

$$F_1 = 0.0033870P_1^2 + 0.856440P_1 + 16.817750$$
$$F_2 = 0.002350P_2^2 + 1.0257600P_2 + 10.029450$$
$$F_3 = 0.0006230P_3^2 + 0.897700P_3 + 23.333280$$
$$F_4 = 0.0007880P_4^2 + 0.851234P_4 + 27.634000$$
$$F_5 = 0.0004690P_5^2 + 0.807285P_5 + 36.856880$$
$$F_6 = 0.0003998P_6^2 + 0.850454P_6 + 30.147980$$

Unit	G_1	G_2	G_3	G_4	G_5	G_6
P_{min}	10	10	35	35	130	125
P_{max}	125	150	225	210	325	315

Line Losses Co-efficient $B_{ij}X10^{-3}$Matrix

0. 140	0. 017	0. 015	0. 019	0. 026	0. 022
0. 017	0. 060	0. 013	0. 016	0. 015	0. 020
0. 015	0. 013	0. 065	0. 017	0. 024	0. 019
0. 019	0. 016	0. 017	0. 071	0. 030	0. 025
0. 026	0. 015	0. 024	0. 030	0. 069	0. 032
0. 022	0. 020	0. 019	0. 025	0. 032	0. 085

Table I shows optimal scheduling of a Six-unit system by GA Method (including transmission losses) with comparison with PSO results reported in [20]. Fig. 1 and Fig. 2 show the convergence graphs for 700 and 800 MW power demands. *The results of best fuel costs shown in the graph of Fig. 1 and Fig. 2 are in Indian Rs (1 $=45 Rs).* The elapsed time of GA processing includes the time required for search process and plotting the graph of convergence. Off course without the graph, the time will be less. Anyway, both PSO and GA algorithms need a lot of efforts to tune parameters to work properly and both are using random initial populations.

TABLE I: The Optimal Scheduling of Generators for Demands of 700 MW and 800 MW.

Unit	$P_D = 700$	$P_D = 700$	$P_D = 800$	$P_D = 800$
P_1	16	27.30096	25	32.67373
P_2	24	15.61244	12	15.81606
P_3	138	120.31087	116	141.66228
P_4	116	116.77564	182	131.31169
P_5	208	226.83767	287	252.37105
P_6	214	212.40501	203	251.55072

P_{Loss}	18	19.2426	26	25.3855
FC ($/h)	821.93	820.42	935.87	931.106
Time (Sec.)	1.2	1.78	1.6	1.74
Method	PSO [21]	GA	PSO [21]	GA

FC=Fuel Cost

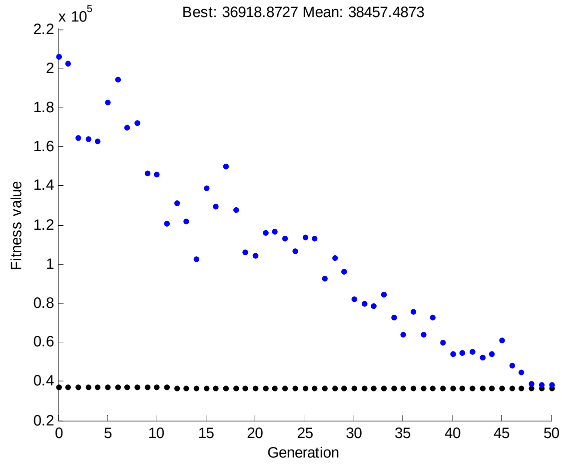

Fig. 1. Convergence graph for 6-units with P_D=700 MW

B. Case II: 40-Units with valve-loading effect (neglect losses).

The data of cost coefficients and coefficients reflecting valve-point effects [21-22] is given below in table II. Losses are neglected for sake of comparisons and the case of load demand of 10,500 MW is considered as in [20-21]. Table III shows self-explanatory numerical results obtained with GA in comparisons with PSO approaches in [21-22].

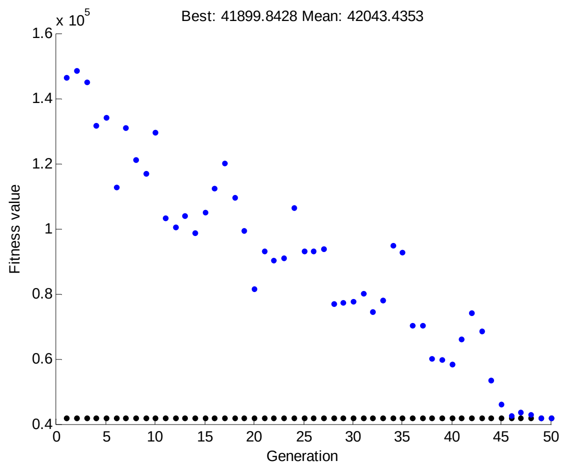

Fig. 2. Convergence graph for 6-units with P_D=800 MW

TABLE II: 40-Units Data (Non-smoothing Fuel Cost)

Unit	$P_{i,min}$	$P_{i,max}$	a_i	b_i	c_i	e_i	f_i
1	36	114	0.00690	6.73	94.705	100	0.084
2	36	114	0.00690	6.73	94.705	100	0.084
3	60	120	0.02028	7.07	309.54	100	0.084
4	80	190	0.00942	8.18	369.03	150	0.063
5	47	9 7	0.0114	5.35	148.89	120	0.077
6	68	140	0.01142	8.05	222.33	100	0.084
7	110	300	0.00357	8.03	287.71	200	0.042
8	135	300	0.00492	6.99	391.98	200	0.042
9	135	300	0.00573	6.60	455.76	200	0.042
10	130	300	0.00605	12.9	722.82	200	0.042
11	94	375	0.00515	12.9	635.20	200	0.042
12	94	375	0.00569	12.8	654.69	200	0.042
13	125	500	0.00421	12.5	913.40	300	0.035
14	125	500	0.00752	8.84	1760.4	300	0.035
15	125	500	0.00708	9.15	1728.3	300	0.035

16	125	500	0.00708	9.15	1728.3	300	0.035
17	220	500	0.00313	7.97	647.85	300	0.035
18	220	500	0.00313	7.95	649.69	300	0.035
19	242	550	0.00313	7.97	647.83	300	0.035
20	242	550	0.00313	7.97	647.81	300	0.035
21	254	550	0.00298	6.63	785.96	300	0.035
22	254	550	0.00298	6.63	785.96	300	0.035
23	254	550	0.00284	6.66	794.53	300	0.035
24	254	550	0.00284	6.66	794.53	300	0.035
25	254	550	0.00277	7.10	801.32	300	0.035
26	254	550	0.00277	7.10	801.32	300	0.035
27	10	150	0.52124	3.33	1055.1	120	0.077
28	10	150	0.52124	3.33	1055.1	120	0.077
29	10	150	0.52124	3.33	1055.1	120	0.077
30	47	97	0.01140	5.35	148.89	120	0.077
31	60	190	0.00160	6.43	222.92	150	0.063
32	60	190	0.00160	6.43	222.92	150	0.063
33	60	190	0.00160	6.43	222.92	150	0.063
34	90	200	0.0001	8.95	107.87	200	0.042
35	90	200	0.0001	8.62	116.58	200	0.042
36	90	200	0.0001	8.62	116.58	200	0.042
37	25	110	0.0161	5.88	307.45	8 0	0.098
38	25	110	0.0161	5.88	307.45	8 0	0.098
39	25	110	0.0161	5.88	307.45	8 0	0.098
40	242	550	0.00313	7.97	647.83	300	0.035

TABLE III: 40 UNITS OPTIMUM DISPATCH

Unit	Ref. [22]	Ref. [23]	Proposed GA	
			Output	Unit fuel
1	114	110.8731	108.76409	918.9593
2	114	111.2066	114.00000	963.00789
3	60	97.4	117.63920	1430.34398
4	190	179.7332	190.00000	2281.39049
5	97	87.9256	97.00000	783.15995
6	140	140	140.00000	1583.69816
7	300	259.6023	300.00000	3045.77548
8	300	284.5999	300.00000	2955.91133
9	290.1619	284.6004	300.00000	2975.59133
10	130	130	136.56586	2598.31611
11	94	168.7999	94.78717	1904.34063
12	94	94	94.38809	1913.60729
13	125	214.7598	127.91692	2581.78318
14	125	304.5196	311.14543	5272.98739
15	394.28	394.2794	282.76897	4910.60871
16	394.28	394.2794	203.20460	3894.29652
17	500	489.2794	500.00000	5466.41285
18	500	489.2795	500.00000	5458.25285
19	550	511.2795	550.00000	6034.26653
20	550	511.2794	550.00000	6034.24653
21	550	523.2794	550.00000	5387.85973
22	550	523.2796	550.00000	5387.85973
23	550	523.2795	550.00000	5370.57973
24	550	523.2794	550.00000	5370.57973
25	550	523.2794	550.00000	5598.19473
26	550	523.2794	550.00000	5598.19473
27	10	10	14.03671	1205.19275
28	10	10	11.97786	1170.08709
29	10	10	11.30362	1159.55107
30	97	89.0624	97.00000	783.15995
31	190	190	190.00000	1523.74843
32	190	190	190.00000	1523.74843
33	190	190	190.00000	1523.74843
34	200	200	200.00000	1917.97937
35	200	172.2847	200.00000	1860.68937
36	200	200	200.00000	1860.68937
37	110	110	107.50147	1136.87092
38	110	110	110.00000	1160.64994
39	110	110	110.00000	1160.64994
40	511.28	511.2794	550.00000	6034.26653
Total	**10,500 MW**	**10,500 MW**	**10,500 MW**	
Total	**124,577.273***	**121,432.177***	**119,732.25**	

It is clear and self-explanatory from table III that the proposed GA has provided better solutions compared with PSO approaches.

Tuned GA Options/Parameters for Case II are Generations: 80, Population size: 500, Crossover rate: 0.8 and Migration rate: 0.1.

*The author has doubt about the total fuel costs of [21] and [22] and believes that the reported fuel costs [21] and [22] should be as Numerals ***in bold italic between brackets*** as shown in last row of table III.

Fig. 3 showed the convergence of the solution obtained by GA. The total of 80 iterations was spent during this process with elapsed time 7.34 seconds.

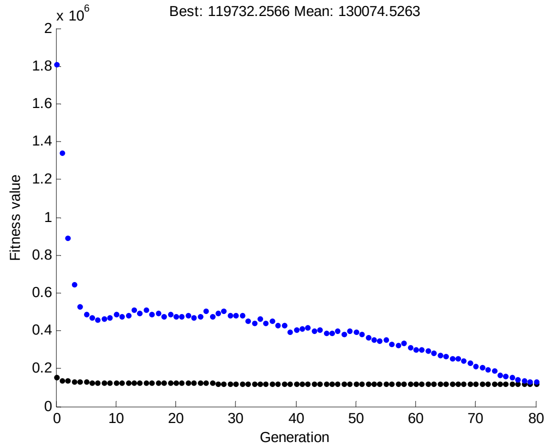

Fig. 3 Solution convergence by GA for 40 units

The best fuel cost results by GA method is much lower than the reported results in [21-22] with smooth convergence as illustrated in Fig. 3.

VI. CONCLUSION

This paper presents an application of GA approach for solving smooth and non-smooth ELD problem with valve-point ripple effects. GA always found solutions with global minimum or even near to global minimum of total fuel costs. These solutions may be changed from run to run as the GA normally uses an initial population randomly. The algorithm had been applied successfully to ELD with considering the effects of valve-point loading while relaxing all other equality and inequality constraints. Main disadvantage of GA, that lot of efforts were exerted to tune parameters by trial and error for better performance. However, GA is searching with population not like Pattern search that normally uses single initial point. However, GA is can be applied simply after formulating the objectives and constraints. In general, GA can provide good performance for Power System Optimization problems.

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