A Practical Solution for Robotic Arm of the Towers of Hanoi Problem

A. S. Zaforullah Momtaz, Md. Fayyaz Khan, Aloke Kumar Saha, and Kazi Shamsul Arefin

Abstract—**Many algorithms are available to solve the puzzle. In this paper we developed and presented the Natural Algorithm (NA). This algorithm not only provides solution for the three towers and eight disks problem but also able to provides solution for multiple towers and disks problem through minimum number of operations. Solution for sixteen towers and fifteen disks has been shown in the paper requiring minimum number of operations.**

Index Terms—**Natural Algorithm (NA), Towers of Hanoi (TOH), N-Tower Solution.**

I. Introduction

There are two main things involved in the TOH puzzle. These are the number of towers and the number of disks. Recently different robots are implemented to perform the puzzle. One of them used a touch screen and 3 touch pins to move the disks from source tower to the destination tower [4]. Another robot is implemented, which have 3 towers at the 3 corners of a triangle and having an arm at the center of the triangle to move the disks [5]. The mentioned robots are able to solve only 3 towers puzzle.

But in this paper, NA can solve the TOH problem with 15 disks and 8 towers. By utilizing 8 towers, the number of movements of the disks is reduced. With reference to Table 1, it can be seen that with 15 disks and 3 towers, 32,767 movements of the robot arm will necessary, where for 8 towers and 15 disks, only 31 movements.

The main portion of Hanoi is a mechanical arm which is designed to capture a disk and to move it from one tower to another tower. At the bottom position of the arm, there is an electromagnet attached to capture the disks as the disks are made of steel. The arm can move in vertical direction with the help of a dc motor and also can move in horizontal direction with the support of a stepper motor [6].

To reduce the complexity of mechanical system a specific location is created for each of the tower on the periphery of a circle. And the arm is placed just at the centre of it i.e. there is 45° angular displacement in between two towers. The arm can move freely in both clock-wise and counter-clock-wise direction.

Manuscript received April 8, 2011; revised July 11, 2011

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II. Background or Origin

Dictionary of Mathematics that the Tower of Hanoi puzzle is of ancient Indian origin [15], as the above rather widespread legend would also suggest. But it seems that it was actually invented from scratch probably together with the accompanying legend only in 1883 by the French mathematician Eduard Lucas [14]. At least there is no known written record about the puzzle or the legend prior to 1883. He was inspired by a legend that tells of a Hindu temple where the pyramid puzzle might have been used for the mental discipline of young priests. Legend says that at the beginning of time the priests in the temple were given a stack of 64 gold disks, each one a little smaller than the one beneath it. Their assignment was to transfer the 64 disks from one of the three poles to another, with one important provisonal large disk could never be placed on top of a smaller one. The priests worked very efficiently, day and night. When they finished their work, the myth said, the temple would crumble into dust and the world would vanish.

Professor Lucas called his invention "Tower of Hanoi". The singular refers to just one tower of disks to be dismantled and reassembled on a different peg. Dictionary and many other sources from the past 25 years use the plural, "Towers of Hanoi". This could reflect a growing belief in the ancient origin of the puzzle (the legend speaks about three towers - one could imagine three pyramidal pagodas with rather thin golden rings placed on their slanting roofs), or it could simply reflect the fact that the language is a living entity whose continuous evolution is impossible to stop. I have done a little survey of the sites listed below, and I found that about 26 of them respect the name given to the puzzle by its inventor and use the singular "Tower of Hanoi", whereas about 29 of them use the plural "Towers of Hanoi".

III. Natural Function

In the puzzle, there are two terms tower and disk. For a specific number of towers the number of disks may vary. It may be above, or below, or equal to the number of towers. Depending on these two variables, different equations are obtained for the natural function. Let, t and d are two variables namely towers and disks respectively. These two are defined as $(0 < t \leq n)$ and $(0 < d \leq m)$ where n and m represents the highest number of the towers and disks respectively. Thus we have the terms t, d & N and t, d > 0.

Using these two variables, the 'Natural' function is implemented to find out the total no. of movements. Here 'm' is considered as 15 (m=15). For this limitation the function is defined as,

$$f(t, d) = \begin{cases} 2^d - 1 & (\text{For, } t = 3) \\ 2d - 1 & (\text{For, } t > d) \\ 4d - 2t + 1 & (\text{For, } t \leq d \leq (t(t-1))/2) \\ 2t^2 - 4t + 9 & (\text{For, } d = (t(t-1))/2 + 1) \\ 20 + 2f(3, p_1) + f(3, p_2) & (\text{For, } (t(t-1))/2 + 1 < d \leq m,\ t = 4) \\ 34 + 2f(3, p_1) + f(3, p_2) & \text{For, } (t(t-1))/2 + 1 < d \leq m - 1,\ t = 5) \\ 58 + 2f(3, p_1) + f(3, p_2) & (\text{For, } d = m,\ t = 5) \end{cases}$$

IV. Natural Algorithm (NA)

Fig.1 shows the the whole electro-mechanical system and fig. 2 contains the photo of the moment in soloving a 5 towers 6 disks puzzle.

Fig. 1. Layout for circuit and other components for the robot.

Fig. 2. Robot designed for solving tower of Hanoi problem.

V. Core Algorithm

NA provides both the number of movements of the disks and the directions of movements of the disks required. Here an additional algorithm is added with NA in order to perform both the mathematical as well as mechanical operations. The operation flow diagram is shown below:

The mechanical portion gets instruction from the core algorithm. Using the natural function 'f(t, d)', (where t stands for towers and d for disks) the problem is classified to solve. The whole problem is classified into some steps as equal to the number of total movements of the given tower-disk combination. NA ensures the minimum movements. Table 1 contains the list of the minimum movements of different combination of towers and disks.

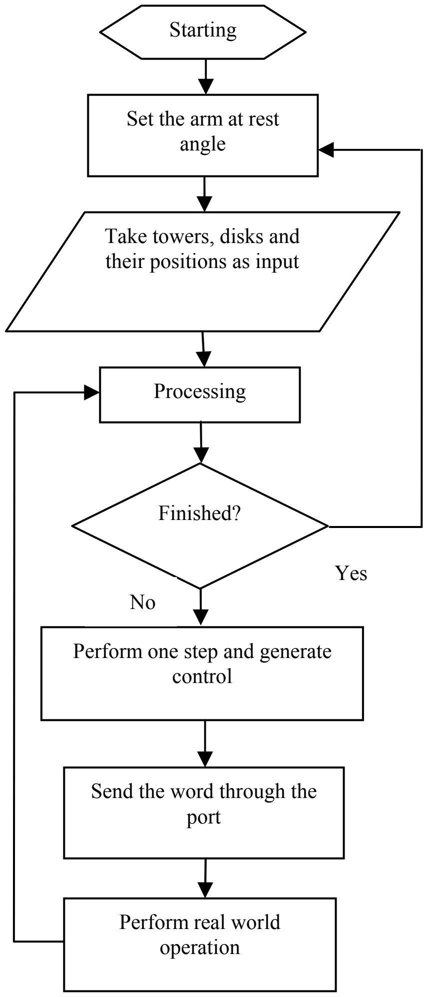

Fig. 3. Flow of operations.

VI. Input and Output System

From real world, the robot collects four inputs. These are,

a) The number of towers in the puzzle,
b) In which places are these placed,
c) The number of disks in the puzzle, and
d) The tower in which these are placed.

On the other hand Han Fig. 3oi provides the output in the real world by moving all the disks from source tower to destination tower using the temporary towers maintaining all the rules.

There are total 8 toggle switches used to detect the towers. The switches are arranged such that when a tower is placed in any of the 8 places the corresponding switch provides logic '1' to notify that a tower is placed at that position otherwise the switch provides logic '0'. The output of these switches is used as the input of the MUX [7]. So for which selection input of MUX the output is high denotes the tower is placed at that position. Again the number of total high output of the MUX provides the total number of towers that are given in the puzzle. Hanoi cannot detect the number of disks that are provided in the puzzle directly. This information is given through the keyboard of the computer system. Also information about source tower is given through keyboard.

VII. Machine Interfacing and Hardware Operation

Hanoi uses total 10 bit information for communication purpose with the host computer system to get the machine instruction and also to understand the system program according to the situation of the real world. It provides only 1 bit input to the computer and the program provides 9 bits of information to the machine. The output of the machine is obtained from a 16 input MUX, but here only 12 bits are used. The inputs of this MUX are the sensors attached at different positions of the arm and body. And the rest eight inputs are obtained from the tower detector toggle switches. Depending on the output of the MUX the system program generates the appropriate control word and then sends it to the machine. After getting this control word it is classified according to the machine requirements. 4 bits of the word is distributed for the DC and stepper motor. 1 bit is allotted to handle the electromagnet attached at the bottom of the arm. Other 4 bits are distributed as the selector input of the MUX. The parallel port (LPT-1) is used to send and receive data for communication in between the robot and the host computer system. Graphically it is as followed,

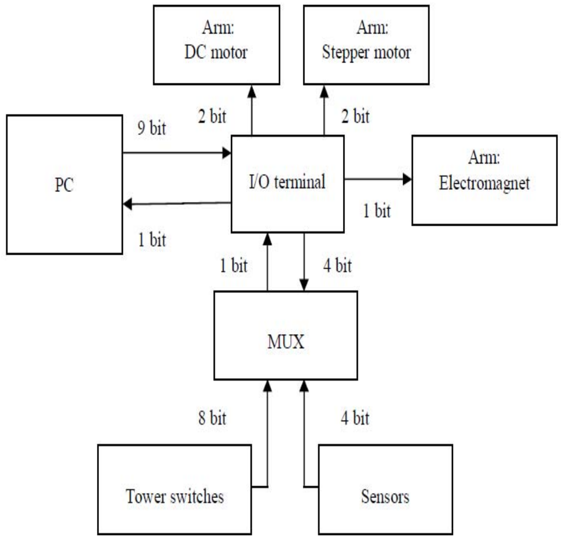

Fig. 4. block diagram of the whole system

TABLE 1: Number Of Movements Of N Towers For 15 Disks.

d \ t	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	3	3	3	3	3	3	3	3	3	3	3	3	3	3
3	7	5	5	5	5	5	5	5	5	5	5	5	5	5
4	15	9	7	7	7	7	7	7	7	7	7	7	7	7
5	31	13	11	9	9	9	9	9	9	9	9	9	9	9
6	63	17	15	13	11	11	11	11	11	11	11	11	11	11
7	127	25	19	17	15	13	13	13	13	13	13	13	13	13
8	255	33	23	21	19	17	15	15	15	15	15	15	15	15
9	511	41	27	25	23	21	19	17	17	17	17	17	17	17
10	1023	49	31	29	27	25	23	21	19	19	19	19	19	19
11	2047	65	39	33	31	29	27	25	23	21	21	21	21	21
12	4095	81	47	37	35	33	31	29	27	25	23	23	23	23
13	8191	113	55	41	39	37	35	33	31	29	27	25	25	25
14	16383	145	63	45	43	41	39	37	35	33	31	29	27	27
15	32767	209	71	49	47	45	43	41	39	37	35	33	31	29

VIII. Processing System

Firstly, the towers are placed at the appropriate positions. Then the disks are placed at the source tower. After placing the disks a command is given that ‘m’ numbers of disks are placed at z^{th} tower position through the keyboard of the computer.

After getting all the information the arm moves along counter clock wise direction to identify the first tower from the rest angle position. Here an optical sensor is used to identify this. After getting it, the central function initializes a two dimensional array to hold the number of disks and towers of the puzzle and also classify the puzzle to solve using the right function. And then send one by one disk movement instruction by setting the exact control word. Normally the arm is placed at the top most position. When a disk is needed to be lifted from source tower it is moved in downward direction. When the top of the disk is obtained, the motion of the arm is stopped. Then the electromagnet is energized to capture the disk. After capture the disk, the arm is moved in the upward direction. When the top most position is obtained the arm is moved to set it on the top of the destination tower of that disk to place it there. Here 2 sensors are used to detect the top and the bottom most position. And another sensor is used to detect the top position of the disk stack and of the surface of the box.

IX. Conclusion

TOH based robots are familiar. But Hanoi is different in some ways. There are total 8 tower positions created. Using any three of these many three towers and maximum 15 numbers of disks combinations can be made. Again there may be one tower gap, i.e. 90° gap in between two towers or more than it. Also 4 or more towers and different number of disks combination can be made in the same way. Due to these huge tower-disk combinations Hanoi should be a nice toy for kids. And also this mechanical system can be applied in industrial purpose.

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