Efficient Hardware Implementation of ESPRIT-like Algorithm for Range Estimation of Chirp Spread Spectrum

Arjumand Yaqoob, Umar Farooq, Ghulam Abbas, and Muhammad Usman Asad

Abstract—for the efficient Range estimation of IEEE 802.15.4a CSS, a new ESPRIT-like algorithm is presented. The conventional ranging algorithms have estimated frequency components from channel frequency response of the received signals. However, the proposed algorithm which is based on the separated multi-band chirp signals can perform range estimation by using the phase relation between sub-chirps in low and high bands. The simulation and implementation results of the proposed algorithm have a gain of about 5 dB and improved range error ratio of 3~5%, respectively, compared to ESPRIT algorithm. The proposed algorithm is verified for hardware implementation using FPGA. The implementation results show that the computational acceleration achieved can be employed for efficient hardware implementation of a Ranging system on chip.

Index Terms—Range estimation, CSS, Ranging System on Chip

I. INTRODUCTION

CSS is the new IEEE Std.P802.15.4a at 2.4GHz as in [1], which uses multi-band chirp signals for data communication. Since the received chirp signals can be transformed to sinusoids by the dechirping process as in [2], super-resolution (SR) algorithms such as ESPRIT, MUSIC and matrix pencil can be employed to estimate the propagation delay of arrival paths in CSS without an additional channel frequency response estimation process as in [4][6]. Unlike general chirp signal based system, CSS transmits the same sub-chirp signal via two different bands, so the dechirped sinusoids of the received sub-chirp signal in high-band have the same frequency as those of the received sub-chirp signal in low-band, but with different phases. This phase difference is a function of the center frequency difference between low and high bands and propagation delay of arrival paths. Therefore, we can extract propagation delay information from the estimated phase difference for range estimation. However, only the first arrival path contains the propagation delay information for the direct path

between transceiver. Therefore, we consider only first arrival path among multi-path in this paper.

II. PRELIMINARY

A. Signal Model for CSS

CSS transmits a full chirp signal which consists of four individual sub chirp signals which use either high or low band. CSS has four piconets to support simultaneous data communication for multi-users. For our convenience without losing generality, we assume that only two sub chirps in the different two bands are used in this paper. Time-frequency relation of two sub chirps in low and high band in piconet1 is shown in Fig.1. The received chirp signals in Fig.1 are delayed by τ_1 with respect to reference signal and cause constant frequency difference between the received and reference CSS signal. The starting time of reference CSS signal in Fig.1 is decided by correlation process by which the receiver can perform coarse estimation for τ_1 . By multiplying the received CSS signal with the conjugated reference CSS signal, the dechirped sinusoids of frequency $-\mu\tau_m$ can be generated as in (1).

Figure 1. Time frequency relation of two sub chirp signals

In the Fig.1, β_μ is the amount of the swept frequency, other parameters are explained below (1):

$$y_k(t) = \sum_{m=1}^d a_m \exp \left[j \left(-\mu\tau_m t - \omega_k \tau_m + \frac{\mu}{2} \tau_m^2 \right) \right] + n'(t) \quad (1)$$

where $k \in \{0,2\}$ and $n'(t)$ is the projection of AWGN onto the sub chirp replica. The ω_k is the center frequency of the k -th sub chirp, while a_m and τ_m denotes m -th path's amplitude and propagation delay with respect to start time of reference signal in the receiver. We can estimate the start time of reference chirp signal which is the coarse start time of the received signal by correlation method between reference and

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received signal. d is the total number of paths and constant μ defines the inclination of sub-chirp signal in time-frequency domain. As shown in (1), the frequency components of dechirped sinusoids are oriented from τ_m . Therefore, the multipath delay of CSS can be estimated from frequency estimation.

B. Conventional ESPRIT Approach

In case of ESPRIT, the received dechirped sinusoids of (1) can be used to form a correlation matrix R_k and EVD can be applied to remove the noise component in (1). Then, we can estimate multi-path component as following:

$$\hat{\mathbf{R}}^{(0)} = \hat{\mathbf{R}}^{(1)}\Phi \text{ where } \Phi = \mathbf{T}^{-1}\Lambda\mathbf{T} \quad (2)$$

and $\Lambda = \text{diag}\left[e^{j(-\mu T_s \tau_1 + \theta_1)}, \dots, e^{j(-\mu T_s \tau_m + \theta_m)}\right]$

where T_s is the sample duration and θ is the induced error due to AWGN noise while calculating Λ . $\hat{\mathbf{R}}^{(0)}$ and $\hat{\mathbf{R}}^{(1)}$ are obtained from R_k matrix by deleting its first and last rows respectively. The above equation can be solved for eigen values Λ_i and from estimated values each of the propagation delay τ_m can be measured. The minimum value of $\ln(\Lambda)/-j\mu T_s$ can be selected to calculate the estimated propagation delay for direct path as following:

$$\hat{\tau}_1 = \frac{\min(\ln(\Lambda))}{-j\mu T_s} = \frac{-\mu T_s \tau_1 + \theta_1}{-\mu T_s} = \tau_1 + \frac{\theta_1}{-\mu T_s} \quad (3)$$

It is evident through above equation that the estimated $\hat{\tau}_1$ would be more robust to noise $n'(t)$ if the divider in above equation becomes more large. If one sub-chirp is composed of N samples we can rewrite (3) as:

$$\hat{\tau}_1 = \tau_1 + \frac{\theta_n}{-\left(\frac{\beta_\mu}{NT_s}\right)T_s} \approx \tau_1 + \frac{\theta_n}{-\left(\frac{|\omega_k - \omega_{k+1}| \times \alpha}{N}\right)} \approx \tau_1 + \eta_{\text{single}} \quad (4)$$

where $\alpha = |\omega_k - \omega_{k+1}|/\beta_\mu = 6.3/8.6875 \approx 0.73$.

Single represents the approximated propagation delay error.

III. PROPOSED ESPRIT APPROACH

The Proposed algorithm can estimate propagation delay τ_m based on the phase relation between dechirped sinusoids of sub chirps in low and high bands. This phase relation can be formulated as following:

$$y'_{k,m}(t) = y'_{k+1,m}(t) \times \exp\left[j\left((\omega_k - \omega_{k+1})\tau_m\right)\right], \text{ for } k \in \{0, 2\} \quad (5)$$

where $y'_{k,m}(t)$ and $y'_{k+1,m}(t)$ are the dechirped sinusoids of m -th path of the noiseless form of $y_k(t)$ and $y_{k+1}(t)$, respectively. If we can estimate this phase $(\omega_k - \omega_{k+1})\tau_m$ denoted by ψ using ESPRIT-like algorithm, then the estimated τ_m would be more robust than the result from (3), since divider of θ_1 in below equation (6) becomes more larger than that of (4):

$$\begin{aligned} \hat{\tau}_m &= \frac{\ln(\Lambda)}{-j(\omega_k - \omega_{k+1})} = \frac{(\omega_k - \omega_{k+1})\tau_m + \theta_1}{\omega_k - \omega_{k+1}} = \tau_m + \frac{\theta_1}{\omega_k - \omega_{k+1}} \\ &= \tau_m + \eta_{\text{double}} \text{ where } \eta_{\text{double}} \approx \alpha \frac{\eta_{\text{single}}}{N} \end{aligned} \quad (6)$$

Double is the approximated delay error associated with ESPRIT like algorithm? In case of ESPRIT, auto-correlation and EVD is performed to separate noise space from signal space. However, the proposed algorithm doesn't use autocorrelation matrix in order to retain the phase information between dechirped sinusoids in low and high bands. Instead of auto-correlation matrix, we use data matrices $\mathbf{Y}_k$ and $\mathbf{Y}_{k+1}$ and joint SVD as following:

$$\begin{bmatrix} \mathbf{Y}_k \\ \mathbf{Y}_{k+1} \end{bmatrix} = \begin{bmatrix} \overset{d}{\hat{\mathbf{U}}_d} & \overset{l-d}{\hat{\mathbf{U}}_d^\perp} \end{bmatrix} \Sigma_r \begin{bmatrix} \overset{d}{\hat{\mathbf{V}}_d} \\ \overset{l-d}{\hat{\mathbf{V}}_d} \end{bmatrix}, \mathbf{Y}_k = \begin{bmatrix} y_k(0) & \dots & y_k(L-1) \\ \vdots & \dots & \vdots \\ y_k(N-L-1) & \dots & y_k(N-1) \end{bmatrix} \quad (7)$$

where d is the number of multipath as in (1) as well as the number of the column vectors which span signal space of $[\mathbf{Y}_k \mathbf{Y}_{k+1}]^T$. After joint SVD, we derive signal space of $[\mathbf{Y}_k \mathbf{Y}_{k+1}]^T$ according to the left d singular values as in [4] and perform TLS-TLS approximation as following:

$$\begin{aligned} \hat{\mathbf{Y}}_k &= \Pi_{\hat{\mathbf{U}}_c} \mathbf{Y}_k \Pi_{\hat{\mathbf{V}}_r} = \hat{\mathbf{U}}_c \hat{\mathbf{U}}_c^* \mathbf{Y}_k \hat{\mathbf{V}}_r \hat{\mathbf{V}}_r^* \\ \hat{\mathbf{Y}}_{k+1} &= \Pi_{\hat{\mathbf{U}}_c} \mathbf{Y}_{k+1} \Pi_{\hat{\mathbf{V}}_r} = \hat{\mathbf{U}}_c \hat{\mathbf{U}}_c^* \mathbf{Y}_{k+1} \hat{\mathbf{V}}_r \hat{\mathbf{V}}_r^* \end{aligned} \quad (8)$$

where $\Pi_{\hat{\mathbf{V}}_r} = \hat{\mathbf{V}}_d \hat{\mathbf{V}}_d^*$ and $\Pi_{\hat{\mathbf{U}}_c} = \hat{\mathbf{U}}_d \hat{\mathbf{U}}_d^*$ denote the projector onto the approximated row space and column space of $[\mathbf{Y}_k \mathbf{Y}_{k+1}]^T$, respectively. $\hat{\mathbf{Y}}_k$ and $\hat{\mathbf{Y}}_{k+1}$ are referred as filtered matrix in this paper. Each data matrix of size L by $N-L$ was made without the knowledge about signal space of dimension d . Therefore, the filtered matrix is also of size L by $N-L$. However, this filtered matrix can not be directly applied to key parts of conventional ESPRIT procedures after (10) due to its size as in [3]. Therefore, we reconstruct the received sample sequence by using the filtered_matrix as in (9) and form a matrix of size $N-d$ by d composed of the reconstructed sequence.

$$\begin{aligned} \hat{y}_k(m) &= \sum_x \sum_y \hat{\mathbf{Y}}_k(x, y) \text{ where} \\ \{(x, y) \mid x + y = m \text{ and } x \in \{0, \dots, L-M-1\}, y \in \{0, \dots, L\}\} \end{aligned} \quad (9)$$

$\hat{\mathbf{E}}_k$ and $\hat{\mathbf{E}}_{k+1}$ denotes the reconstructed matrix using the low and high bands of sub chirps, respectively.

$$\hat{\mathbf{E}} = [\hat{\mathbf{E}}_k \mid \hat{\mathbf{E}}_{k+1}] \text{ where } \hat{\mathbf{E}}_k = \begin{bmatrix} \hat{y}_k(0) & \dots & \hat{y}_k(d-1) \\ \vdots & \dots & \vdots \\ \hat{y}_k(N-d-1) & \dots & \hat{y}_k(N-1) \end{bmatrix} \quad (10)$$

The last step below is similar to conventional ESPRIT algorithm as in [5-6]. We try to find a matrix $\mathbf{F} = \begin{bmatrix} \mathbf{F}_k \\ \mathbf{F}_{k+1} \end{bmatrix}$ to

minimize $\mathbf{V} = \left\| \begin{bmatrix} \hat{\mathbf{E}}_k & \hat{\mathbf{E}}_{k+1} \end{bmatrix} \mathbf{F} \right\|_F^2$ subject to $\mathbf{F} \times \mathbf{F} = \mathbf{I}$. It is easily shown that $\mathbf{F}$ is the matrix of right singular vectors of $\begin{bmatrix} \hat{\mathbf{E}}_k & \hat{\mathbf{E}}_{k+1} \end{bmatrix}$ corresponding to the d smallest singular values as in [3]. The phase difference in (5) can be found as the eigen-values of $-\mathbf{F}_k \mathbf{F}_{k+1}^{-1}$.

IV. RANGING ESTIMATION AND IMPLEMENTATION RESULTS

The proposed algorithm has been verified through simulations and implementation results. The simulations are performed for AWGN channel and experimental results for implementation have been drawn for measuring the distance between transceivers of CSS. The proposed algorithm is compared with single and double BW ESPRIT as shown in Fig.2 and 3. Actually the single BW ESPRIT is based on one sub chirp of CSS, while double BW ESPRIT is based on the continuous chirp signal which uses full bandwidth in contrast with CSS signal.

Figure 2. The simulation results in AWGN channel

Figure 3. Experimental results in indoor environment

Figure 4. Ranging system block diagram

In Fig.2, the mean and STD of ranging errors are compared and the range errors are mapped according to measured distances between transceivers in indoor geo location in Fig.3.

In Fig.2 and 3 the vertical line shows the STD of the range error while the centre of vertical line is the mean value for measurements. The simulation results can embark the superiority of the algorithm over ESPRIT by achieving the SNR gain of about 5dB, compared to the double BW ESPRIT. Similarly for the ratio of error to measured distances as mapped in Fig.3 shows less error ratio about 3~5% for proposed algorithm as compared to double BW ESPRIT.

In Fig.4 the block diagram of the implemented system used for ranging estimation is shown. The system comprises of a base band FPGA board by Altera Startix-II EP2S180 and two RF transceiver MAX2830 EV KITs. This implemented system can transmit and process various chirp signals for range estimation. The chirp signal is stored in FPGA board and transmitted by RF module, while the received chirp signal is detected for range estimation by the hardware part of the board. NIOSII core of the board control the hardware part and passed the received samples to the PC whereas the high resolution algorithm the ESPRIT like algorithm has been implemented in part by employing the hardware and software blocks implemented using Simulink and the Singular value decomposition (SVD) part in hardware on FPGA.

V. CONCLUSIONS

In this paper, we proposed and implemented a new ESPRIT-like algorithm for efficient range estimation. The algorithm can be used for range estimation using CSS signal. This algorithm utilized the CSS sub-chirp signals in different two bands more efficiently than ESPRIT and the hardware implementation verifies this concept to be used in efficient ranging systems. Therefore, we expect that this algorithm can be applied to ranging system which uses multi-band chirp signals. In this regard an efficient ranging system can be built using CSS.

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