The d-q Modeling, Order Reduction and Numerical Analysis of Double Cage Three Phase Induction Machine for Power System Studies

Sudhir Kumar, P. K. Ghosh and S. Mukherjee

Abstract—The study of induction machine is becoming important in power system dynamic analysis. An accurate algorithm for determining induction machine transients is presented here. Higher order model of double cage induction machine is simplified and solved using Runge-Kutta, Trapezoidal and Euler's methods in Mat-lab platform. The authors compute the rotor speed versus time profile, stator current and stator flux linkage for various load conditions and supply voltage disturbances using the concept of the speed-build up of induction machines. The 7th order model of a double cage induction machine has been reduced here to a 5th order model with a structure different from the regular 5th order model for single cage induction machine. Model Order Reduction (MOR) incorporates a set of techniques aiming at generating reduced order models with lower complexity while ensuring the I/O response and other characteristics of the original model (such as passivity). To obtain stability in digital simulation, winding flux-linkages are used as state variables in contrast to current flow. Comparison of computational times in the reported three methods is made. We have verified less computation time in reduced order models and an improvement in the accuracy is also observed.

Index Terms— Induction machine, Order reduction, Stability, Transient analysis, Power System

I. INTRODUCTION

The squirrel cage induction machine drives in power systems are regarded as the state of the art of industrial variable speed applications. They represent the best solution for the substitution of dc machine drives which require thorough and expensive maintenance. The modeling of complex dynamic systems is one of the most important subjects in the field of engineering. A model sometimes happens to be too complicated to be useful in real problems. So, approximation procedures based on physical considerations or mathematical approaches are used to obtain simpler models of the original one. The subject of model reduction is very important and relevant to engineers and scientists working in many fields of engineering, especially, for those who work in the area of process control and power systems. Effort towards obtaining lower order models from higher order model systems aims at deriving stable reduced

order models from the stable original ones and assuring that the reduced order model matches some quantities of original one. The past three decades have witnessed many researches in state space and transfer function forms towards these two objectives [1-31]. Further, high power induction motors have found increasing use in recent years. In order to limit the starting current and to have a reasonably large starting torque, these motors are generally equipped with a double-cage or deep-bar rotor. In this paper, the machine models are developed for a double-cage induction machine. The resulting models can be applied with some degree of approximation to induction machines with deep-bar rotor as well. This is because an equivalent double-cage machine can always be found with an admittance locus closely fitting to that of a deep-bar induction machine [18].

Modeling each poly-phase winding effectively as second-order d-q equation in the generalized theory of electrical machines, the transient model of a double cage induction machine comes out a seventh-order with six electrical variables and one mechanical variable, i.e. motor speed, as state variables. But, the complete representation of induction motors in the literature [18-20] is usually described by a fifth-order model whereas it is generally accepted to include induction machine models in certain power system studies [22-25]. However, in those cases it is worth considering that, the induction motors are usually of the larger size. The fifth-order model or its derivatives to represent these large machines would generally ignore the presence of the second rotor cage winding. It should be noted that for the transient stability studies in power systems, it has become customary to use a simplified version of this fifth-order model by neglecting stator flux transients.

The importance of the starting cage of high power motors in system analysis has been worked out in the research papers [28-29] in which order reduction of the complete seventh-order model were achieved by neglecting stator transients. Consequently, they have neglected the speed voltages in both rotor windings incorrectly resulting in a poor agreement of the speed behavior as compared to that of the full-order model. Moreover, the results would only be applicable in cases where high frequency components in the power flow are not of interest such as in transient stability studies.

In this study, we will show that the full-order model of an induction motor with double-cage rotor can be readily reduced to a fifth-order model while practically retaining the instantaneous behavior of the rotor speed, stator current and stator flux linkage at the machine terminals. The organization

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of this paper is as follows. Section II describes the analytical details of full order machine model. The reduced order machine model is detailed in section III. The resulting equations neglecting faster transients are also derived. The methodology for machine order reduction is presented with flow chart diagram in Fig.1 of section IV. Section V portrays numerical results and discussions by using Runge-Kutta, Trapezoidal and Euler's methods in Matlab platform. Finally, we conclude in section VI.

II. FULL ORDER MACHINE MODEL

For the modeling of induction machines, we choose the flux linkage as the state variable. Flux linkages are used since in the digital simulation of the model, the variables provide more stability with the same integration time step. Equivalently, a larger time step can be used to obtain the same accuracy; thus requiring less computational time.

This is the fact that flux linkages of electromagnetically coupled circuits cannot change instantaneously, whereas circuit currents which make up the flux linkages can relatively change slowly rather than abruptly in cases where the circuits are singly fed, such as considered here. This is particularly true if the fast transients due to an electromagnetically coupled circuit are neglected and computational instability can even occur which has been observed in the simulation studies.

Following the aforesaid assumptions as in the generalized theory of electrical machines [26] and using a synchronously rotating reference frame, the electrical equations in complex form for the stator, inner and outer cage windings are respectively given by

$$\begin{aligned} v_1 &= i_1 r_1 + j\omega \psi_1 + \frac{d\psi_1}{dt} \\ 0 &= i_2 r_2 + j\omega \psi_2 + \frac{d\psi_2}{dt} \\ 0 &= i_3 r_3 + j\omega \psi_3 + \frac{d\psi_3}{dt} \end{aligned} \quad (1)$$

where, the voltage (v), currents (i) and flux-linkages (Ψ) are time dependent complex variables and related to the real d-q variables by

$$f = f_d - jf_q \quad (2)$$

Here, f represents a voltage, current or flux-linkage. Employing the per-unit system with equal per-unit mutual inductances between the three windings on each axis, the inductance matrix L given by

$$\psi = Li \quad (3)$$

can be written as

$$L = \begin{bmatrix} l_m + l_1 & l_m & l_m \\ l_m & l_m + l_2 & l_m \\ l_m & l_m & l_m + l_3 \end{bmatrix} \quad (4)$$

Thus the current in matrix form can be written as

$$i = L^{-1} \psi \quad (5)$$

with

$$L^{-1} = \frac{1}{l} \begin{bmatrix} l_{11} & -l_3 l_m & -l_2 l_m \\ l_3 l_m & l_{22} & -l_1 l_m \\ -l_2 l_m & -l_1 l_m & l_{33} \end{bmatrix} \quad (6)$$

where,

$$l_{11} = l_2 l_3 + l_2 l_m + l_2 l_m$$

$$l_{22} = l_1 l_3 + l_1 l_m + l_3 l_m$$

$$l_{33} = l_1 l_2 + l_1 l_m + l_2 l_m$$

$$l = l_1 l_2 l_3 + l_1 l_2 l_m + l_1 l_3 l_m + l_2 l_3 l_m \quad (7)$$

Substitution of (5) in (1) gives the machine equations in state-variable form as

$$\begin{aligned} \frac{d\psi_1}{dt} &= \left[\left(\frac{r_1 l_{11}}{l} \right) + j\omega \right] \psi_1 + \left(\frac{r_1 l_3 l_m}{l} \right) \psi_2 + \left(\frac{r_1 l_2 l_m}{l} \right) \psi_3 + v \\ \frac{d\psi_2}{dt} &= \left[\left(\frac{r_2 l_3 l_m}{l} \right) \psi_1 - \left[\left(\frac{r_2 l_{22}}{l} \right) + j\omega \right] \psi_2 + \left(\frac{r_2 l_1 l_m}{l} \right) \psi_3 \right] \\ \frac{d\psi_3}{dt} &= \left[\left(\frac{r_3 l_2 l_m}{l} \right) \psi_1 + \left(\frac{r_3 l_1 l_m}{l} \right) \psi_2 - \left[\left(\frac{r_3 l_{33}}{l} \right) + j\omega \right] \psi_3 \right] \end{aligned} \quad (8)$$

The equations dealing the mechanical motion in per-unit are given as

$$\frac{ds}{dt} = -\left(\frac{1}{2H\omega} \right) (te - t_l) \quad (9)$$

where, $t_e = \text{Im}(\psi_1^* i_1)$ which can be written with the help of (5) as

$$t_e = \left(\frac{l_m}{l} \right) \text{Im} \{ l_3 \psi_1 \psi_2^* + l_2 \psi_1 \psi_3^* \} \quad (10)$$

For digital simulation, the complex variables in (8) and (10) are resolved into their real d-q components as expressed by (2) resulting in

$$\begin{aligned} \frac{d\psi_{1d}}{dt} &= -\left(\frac{r_1 l_{11}}{l} \right) \psi_{1d} - \omega \psi_{1q} + \left(\frac{r_1 l_3 l_m}{l} \right) \psi_{2d} + \left(\frac{r_1 l_2 l_m}{l} \right) \psi_{3d} + v_d \\ \frac{d\psi_{1q}}{dt} &= \omega \psi_{1d} - \left(\frac{r_1 l_{11}}{l} \right) \psi_{1q} + \left(\frac{r_1 l_3 l_m}{l} \right) \psi_{2q} + \left(\frac{r_1 l_2 l_m}{l} \right) \psi_{3q} + v_q \\ \frac{d\psi_{2d}}{dt} &= \left(\frac{r_2 l_3 l_m}{l} \right) \psi_{1d} - \left(\frac{r_2 l_{22}}{l} \right) \psi_{2d} - s\omega \psi_{2q} + \left(\frac{r_2 l_1 l_m}{l} \right) \psi_{3d} \\ \frac{d\psi_{2q}}{dt} &= \left(\frac{r_2 l_3 l_m}{l} \right) \psi_{1q} + s\omega \psi_{2d} - \left(\frac{r_2 l_{22}}{l} \right) \psi_{2q} + \left(\frac{r_2 l_1 l_m}{l} \right) \psi_{3q} \\ \frac{d\psi_{3d}}{dt} &= -\left(\frac{r_3 l_2 l_m}{l} \right) \psi_{1d} + \left(\frac{r_3 l_1 l_m}{l} \right) \psi_{2d} - \left(\frac{r_3 l_{33}}{l} \right) \psi_{3d} - s\omega \psi_{3q} \\ \frac{d\psi_{3q}}{dt} &= \left(\frac{r_3 l_2 l_m}{l} \right) \psi_{1q} + \left(\frac{r_3 l_1 l_m}{l} \right) \psi_{2q} + s\omega \psi_{3d} - \left(\frac{r_3 l_{33}}{l} \right) \psi_{3q} \end{aligned} \quad (11)$$

$$\frac{ds}{dt} = -\left(\frac{1}{2H\omega}\right)\left[\left(\frac{l_3 l_m}{l}\right)\psi_{1d}\psi_{2q} - \left(\frac{l_3 l_m}{l}\right)\psi_{1q}\psi_{2d} + \left(\frac{l_2 l_m}{l}\right)\psi_{1d}\psi_{3q} - \left(\frac{l_2 l_m}{l}\right)\psi_{1q}\psi_{3d} - t_l\right] \quad (12)$$

The phase voltages V_a , V_b and V_c in matrix form becomes

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = V_m \begin{bmatrix} \sin \theta \\ \sin (\theta - 2\pi/3) \\ \sin (\theta + 2\pi/3) \end{bmatrix} \quad (13)$$

III. REDUCED ORDER MACHINE MODEL

A. Neglecting Faster Transients

This is a normal practice that the outer cage of double-cage induction motor is generally designed to give a limited starting current with a relatively large starting torque. This is accomplished by making the resistance and leakage inductance of the outer (starting) cage higher and lower, respectively than the corresponding parameter of the inner (running) cage resulting in $r_3 \gg r_2$ and $l_3 < l_2$.

A close look into the transient behavior caused by the presence of the two cages is going through by comparing the two time constants of the equivalent rotor winding, $\tau_2 = l_2/r_2$ and $\tau_3 = l_3/r_3$. Since $\tau_3 \ll \tau_2$, the transients associated with the outer cage are much faster than those with the inner cage, as confirmed by an eigen-value analysis [23].

The order-reduction of the motor model is obtained by neglecting the fast transients due to the outer cage. This amounts to setting the derivative of flux-linkage Ψ_3 of the outer cage in the differential equations equal to zero so that Ψ_3 can be algebraically expressed in terms of the other flux-linkages. This value when substituted in the remaining differential equations reduces the system order.

B. Fifth-Order Model

Following the procedure outlined in the preceding section, setting $d\Psi_3/dt=0$ in the last equation of (8) provides

$$\Psi_3 = \frac{\frac{l_2 l_3}{l_{33}}(1-j s')\Psi_1 + \frac{l_1 l_m}{l_{33}}(1-j s')\Psi_2}{1+s'^2} \quad (14)$$

where,

$$s' = (\omega l_3 / r_3 l_{33}) s \quad (15)$$

Substitution of this expression in the first two equations of (8) and in (9) yields the reduced order form of the machine model.

$$\frac{d\Psi_1}{dt} = [(R_{11} + jL_{11})\Psi_1 + (R_{12} + jL_{12})\Psi_2 + v]$$

$$\frac{d\Psi_2}{dt} = [(R_{21} + jL_{21})\Psi_1 + (R_{22} + jL_{22})\Psi_2]$$

$$\frac{ds'}{dt} = \left(\frac{1}{2H\omega}\right)\left[\left(\frac{\omega l}{r_1 r_3 l_{33}}\right)\text{Im}\{R_{12} - jL_{12}\}\Psi_1\Psi_2^* - \left(\frac{\omega l_2}{r_1 r_3 l_{33}}\right)L_{12}\Psi_1^2 - t_l'\right] \quad (16)$$

with

$$R_{11} = - (r_1 l_{11}/l) + (r_1 l_2^2 l_m^2 / l_{33}) / (1+s'^2)$$

$$R_{22} = - (r_2 l_{22}/l) + (r_2 l_1^2 l_m^2 / l_{33}) / (1+s'^2)$$

$$R_{12} = (r_1 l_3 l_m / l) + (r_1 l_1 l_2 l_m^2 / l_{33}) / (1+s'^2)$$

$$R_{21} = (r_2 / r_1) R_{12}$$

$$L_{11} = -\omega - (r_1 l_2^2 l_m^2 / l_{33}) / (1+s'^2)$$

$$L_{22} = - (r_3 l_{33}/l)s' - (r_2 l_1^2 l_m^2 / l_{33}) s' / (1+s'^2)$$

$$L_{12} = - (r_1 l_1 l_2 l_m^2 / l_{33}) s' / (1+s'^2)$$

$$L_{21} = (r_2 / r_1) L_{12}$$

$$t_l' = (\omega l / r_3 l_{33}) t_l \quad (17)$$

This reduced order model holds for any value of motor speed since the basis for it, as seen from the eigen value analysis, is valid for the entire range of motor speed.

For digital simulation purposes, the equations are written in the real form

$$\frac{d\Psi_{1d}}{dt} = R_{11}\Psi_{1d} + L_{11}\Psi_{1q} + R_{12}\Psi_{2d} + L_{12}\Psi_{2q} + V_d$$

$$\frac{d\Psi_{1q}}{dt} = L_{11}\Psi_{1d} + R_{11}\Psi_{1q} - L_{12}\Psi_{2d} + R_{12}\Psi_{2q} + V_q$$

$$\frac{d\Psi_{2d}}{dt} = R_{21}\Psi_{1d} + L_{21}\Psi_{1q} + R_{22}\Psi_{2d} + L_{22}\Psi_{2q}$$

$$\frac{d\Psi_{2q}}{dt} = -L_{21}\Psi_{1d} + R_{21}\Psi_{1q} - L_{22}\Psi_{2d} + R_{22}\Psi_{2q}$$

$$\frac{ds'}{dt} = -\left(\frac{1}{2H\omega}\right)\left[\left(\frac{\omega l}{r_1 r_3 l_{33}}\right)R_{12}(\Psi_{1d}\Psi_{2q} - \Psi_{1q}\Psi_{2d}) - \left(\frac{\omega l}{r_1 r_3 l_{33}}\right)L_{12}(\Psi_{1d}\Psi_{2d} + \Psi_{1q}\Psi_{2q}) - \left(\frac{\omega l_2}{r_1 r_3 l_{33}}\right)L_{12}(\Psi_{1d}^2 + \Psi_{1q}^2) - t_l'\right] \quad (18)$$

The symbols of the above equations are defined in appendix of this paper.

It is to be noted that the reduced order model (18) and the conventional complete model for a single-cage motor are of the same order; but their structures are quite different so that no parameter values for the conventional fifth-order model can be found which provide the same transient behavior as in the double-cage motor.

IV. METHODOLOGY FOR MACHINE ORDER REDUCTION

Figure 1: Flow chart of simulation algorithm

V. RESULTS AND DISCUSSIONS

The reduced order model of double cage induction machine developed was validated by digital simulation and comparison with the response of full order model. The reduced order model is applied to a 4500 HP, 2900 rpm,

6.6Kv, 50 Hz double cage induction machine and its performance characteristics for stator current, stator flux-linkage and rotor speed are obtained as shown in figures 2(a-c) and 3(a-c) under load and no-load torque disturbances, and supply voltage variations. The results are compared with the full order model. The performance characteristic of reduced order model comes out to be almost identical to that of full order model of double cage induction machine. The machine parameters for double cage induction machine under analysis are given in Table I.

The suitable numerical techniques; Runge-Kutta, Trapezoidal's and Euler's methods are used to solve the non-linear differential equations to integrate the differential equation with an integration size of 0.001. The computational time obtained for full order and reduced order models using three numerical methods are also compared in table II. The algorithm presented in this paper introduces a considerable advantage in computational speed as indicated by comparative results given in the table II.

TABLE I. SPECIFICATION AND MACHINE PARAMETERS OF DOUBLE CAGE INDUCTION MACHINE

4500 HP, 2900 RPM, 6.6KV, 50Hz, THREE PHASE INDUCTION MACHINE	
Machine Parameter	P.U. Value
Stator resistance (r_1)	7.4×10^{-3}
Inner cage winding resistance (r_2)	7.27×10^{-3}
Outer cage winding resistance (r_3)	7.36×10^{-3}
Stator inductance (l_1)	3.47×10^{-4}
Inner cage inductances (l_2)	3.58×10^{-4}
Outer cage inductances (l_3)	3.63×10^{-4}
Magnetising inductances (l_m)	0.0115
Inertia constant (h)	2 sec

TABLE II. COMPARISON OF COMPUTATIONAL TIME OBTAINED FOR FULL ORDER AND REDUCED ORDER MODELS IN THREE NUMERICAL METHODS USING MATLAB

Runge-Kutta (RK4) Solver	
Model	Computation Time(ms)
Full order model	503
Reduced order model	271
Trapezoidal Solver	
Model	Computation Time(ms)
Full order Model	600
Reduced order model	358
Euler Solver	
Model	Computation Time(ms)
Full order Model	411
Reduced order model	222

Figure: 2 (a)

Figure: 2 (b)

Figure:3(c)

Figure 3(a-c): Response of Full Order Model & Reduced order Model under **no load condition** and at rated supply voltage

Figure: 2 (c)

Fig2 (a-c): Response of Full order Model & Reduced order model under **load condition** and **supply voltage changes (15%)**

Figure: 3 (a)

Figure:3 (b)

VI. CONCLUSION

The simulation results show that the fifth order model is almost identical with the complete seventh order model. The reduced order model can also be used for computing the induction machine contribution to initial power system fault currents including dc offsets, and in selecting and coordinating protective devices. In short, the fifth-order machine model can be substitute for the seventh-order model in power system for any simulation kind of study. The computational times of the system using three numerical methods due to Runge-Kutta, Trapezoidal and Euler have been compared and significant reduction in computation time has been observed.

APPENDIX

$1,2,3$	Subscripts referring to stator, running (inner) and starting (outer) cage windings, respectively.
d, q	Subscript referring to d and q axis.
j	Imaginary unit.
$*$	Superscripts referring to complex conjugates.
ω	Angular frequency of power supply in radian per sec.
v	Terminal voltage of stator winding.
R_j	Resistance of winding j.
l_1, l_2, l_3	Leakage reactance of stator, inner and outer cage windings, respectively.
l_m	Magnetizing inductance in per unit.
S	Motor slip.
ω_e	Angular motor speed in electrical radian per sec.
t_e, t_l	Motor and load torque in per unit.
H	Inertia Constant of motor and load in second.

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