Averaged Modeling of Non-ideal Boost Converter Operating in DCM

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Abstract—The averaged model of non-ideal Boost converter operating in discontinuous mode (DCM) circuit was studied based on the switching elements model, the equivalent circuit method, and the energy conservation law, in which the duty ratio constraint was introduced to improve the model. The open-loop transfer function was derived to build the small signal model, by adding a voltage-controlled feedback loop, the closed-loop transfer function was also derived. The simulation results showed the correctness of the model at last.

Index Terms—Boost converter, Non-ideal, Closed-loop control

I. INTRODUCTION

A general power switching regulation system includes power circuit (viz. switching converter) and control circuit. Both parts interact and work together. There are many kinds of control modes in power switching regulation system, such as voltage-mode control, current-mode control, current-lag-loop-mode control, charge-mode control and so on. Voltage control circuit is simple in structure, and its anti-interference ability is strong, so it is suitable for the situations which don't require high control performance of the circuit[1-3].

There is an assignable question in switch converter modeling that is the deviation between the traditional ideal model and actual circuit[4-6]. Therefore, it is very necessary to study converter switch modeling method considering the non-ideal situation.

In this paper, take Boost converter for example, we study the average circuit model of non-ideal basic converter operating in discontinuous conduction mode (DCM), and use the duty ratio constraint to improve the average model and derive transfer function, then derive the transfer function of closed-loop control model, combining with voltage control feedback loop. Both were simulated under the MATLAB tools to verify the correctness of this model.

II. LARGE SIGNAL AVERAGE MODEL OF NON-IDEAL BOOST CONVERTER IN DCM

A. Large Signal Averaged Modeling of Non-ideal Boost Converter in DCM

When we consider the parasitic parameters of the Boost converter, we can see power switch MOSFET as that ideal switch and on resistance R_{on} are in series, see diode as the series of ideal switch, forward voltage drop V_F and on resistance R_F . R_L , R_c are respectively equivalent series resistance (ESR) of filter inductance and filter capacitor. According to the energy conservation principle, the parasitic parameters of the two switch inductance are equivalent to the inductance branch. And then use the method of average model of switching element, using current-controlled current source instead of active switching elements S, using voltage-controlled voltage source instead of passive switching elements D. So, we can get the large signal average model, shown in Figure 1.

Fig. 1. Large signal average model of non-ideal Boost converter in DCM

B. Duty Ratio Constraint

The duty cycle formula which is based on voltage-second of inductance voltage can not reflect accurately the frequency characteristics of the circuit, and it has limitation. This paper uses the new duty ratio constraint of Boost converter, which was derived in literature [1].

$$d_2 = \frac{2L\bar{i}_L}{d_1 T_s V_{in}} - d_1 \quad (1)$$

The variable was analyzed to direct components and AC small-signal components. Then the duty ratio constraint is

$$d_2 = \frac{2L\bar{i}_L}{d_1 T_s \hat{v}_i} - d_1 = \frac{2L(\bar{I}_L + \hat{i}_L)}{(D_1 + \hat{d}_1)(V_i + \hat{v}_i)T_s} - (D_1 + \hat{d}_1) \quad (2)$$

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$$\frac{d_1 \bar{i}_L}{d_1 + d_2} = \frac{d_1^2 T_s v_i}{2L} = \frac{(D_1 + \hat{d}_1)^2 (V_i + \hat{v}_i) T_s}{2L} \quad (3)$$

$$d_1 \bar{v}_{ap} + (1 - d_1 - d_2) \bar{v}_{ap} = (D_1 + \hat{d}_1) (V_{ap} + \hat{v}_{ap}) + \left[1 - \frac{2L(I_L + \hat{i}_L)}{(D_1 + \hat{d}_1)(V_i + \hat{v}_i)T_s} \right] (V_{zp} + \hat{v}_{zp}) \quad (4)$$

C. AC Small-Signal Model

If we suppose that, the small-signal component is much less than the direct component, make the direct component is zero, and neglect the secondary small-signal product term. Then the (3) and (4) can be predigested to the (5) and (6). So we can get the small-signal linear equivalent model of non-ideal Boost converter in discontinuous conduction model, shown in fig.2.

$$\begin{aligned} \frac{d_1 \bar{i}_L}{d_1 + d_2} &= \frac{d_1^2 T_s v_i}{2L} = \frac{(D_1 + \hat{d}_1)^2 (V_i + \hat{v}_i) T_s}{2L} \\ &= a_1 \hat{v}_i + a_2 \hat{d}_1 \\ d_1 \bar{v}_{ap} + (1 - d_1 - d_2) \bar{v}_{ap} &= -(D_1 + \hat{d}_1) (V_o + \hat{v}_o) + \\ &\left[1 - \frac{2L(I_L + \hat{i}_L)}{(D_1 + \hat{d}_1)(V_i + \hat{v}_i)T_s} \right] \\ &\left[(V_i - V_E - V_o - I_L R_E) + (\hat{v}_i - \hat{v}_o - R_E \hat{i}_L) \right] \\ &= b_1 \hat{v}_i + b_2 \hat{d}_1 + b_3 \hat{i}_L + b_4 \hat{v}_o \end{aligned} \quad (5)$$

Fig.2. small-signal model of Boost converter in DCM

Form fig.2, we can calculate the input-to-output transfer function $G_{vi}(s)$ and the control-to-output transfer function $G_{vd}(s)$ are

$$G_{vi}(s) = \frac{\hat{v}_o(s)}{\hat{v}_i(s)} \Big|_{\hat{d}_1(s)=0}$$

$$\begin{aligned} &= \frac{1 - b_1 - a_1(sL + R_E + b_3)}{1 + b_4 + \frac{(sL + R_E + b_3)(sR_C C + sRC + 1)}{R(sR_C C + 1)}} \\ &= \frac{t_1 t_4}{t_2 t_3 + t_0 t_4} \cdot \frac{(1 + s/w_{z1})(1 + s/w_{z2})}{1 + \frac{s}{Qw_0} + \left(\frac{s}{w_0}\right)^2} \end{aligned} \quad (7)$$

$$\begin{aligned} G_{vd}(s) &= \frac{\hat{v}_o(s)}{\hat{d}_1(s)} \Big|_{\hat{v}_i(s)=0} \\ &= -\frac{b_2 + a_2(sL + R_E + b_3)}{1 + b_4 + \frac{(sL + R_E + b_3)(sR_C C + sRC + 1)}{R(sR_C C + 1)}} \\ &= \frac{t_4 t_5}{t_2 t_3 + t_0 t_4} \cdot \frac{(1 + s/w_{z2})(1 - s/w_{z3})}{1 + \frac{s}{Qw_0} + \left(\frac{s}{w_0}\right)^2} \end{aligned} \quad (8)$$

The parameters of the actual Boost converter as following: input-voltage $V_i = 5V$, output-voltage $V_o = 10V$, Load current $I_o = 0.2A$, $R = 50\Omega$, $L = 19.2\mu H$, $R_L = 3.6m\Omega$, $C = 1000\mu F$, $R_C = 2m\Omega$. The model number of the MOSFET which we used in this paper is 2SK2690. $R_{on} = 10m\Omega$, for Schottky diode $R_F = 1m\Omega$, switching frequency $f_s = 50kHz$, PWM outputs the Peak voltage is $V_m = 1V$. Then there parameters are put into the formulas as following

$$R_E = \frac{4}{3(D_1 + D_2)} \left(R_L + \frac{D_1 R_{on}}{D_1 + D_2} + \frac{D_2 R_F}{D_1 + D_2} \right)$$

$$V_{ap} = -V_o$$

$$D_2 = \frac{2LI_L}{D_1 T_s V_i} - D_1 \quad \text{and} \quad I_L = \frac{I_o}{D_2}$$

R_E is the total equivalent resistance of the three parasitic resistances which were calculated to branch of inductance. Then we can get $D_1 \approx 0.35$, $R_E \approx 22.69m\Omega$, $V_E \approx 0.184V$, $I_L \approx 0.43A$.

Put the four parameters into formula (7) and (8), then we can get formula (9) and (10).

$$G_{vi}(s) = 2.112 \times \frac{(1 + \frac{s}{500000})(1 - \frac{s}{913333})}{1 + \frac{s}{58.42} + \left(\frac{s}{4502}\right)^2} \quad (9)$$

$$G_{vd}(s) = 23.696 \times \frac{(1 + \frac{s}{500000})(1 - \frac{s}{307433})}{1 + \frac{s}{58.42} + \left(\frac{s}{4502}\right)^2} \quad (10)$$

III. MODELLING OF BOOST CONVERTER WITH VOLTAGE FEEDBACK-LOOP CONTROL

Non-ideal Boost converter with voltage-mode control is shown in fig.3. Control circuit is constituted by the controller, the PWM comparator, the clock circuit and the trigger. The circuit takes a sample of the output voltage, and then the sample signal as the feedback signal is put into the input, to make the input voltage stable.

Fig.3. System structure of Boost converter with feedback-loop

A. Sample Circuit and the Pulse Width Modulator

The transfer function of the sample circuit is $H(s)$, and $H(s)v_o \approx v_{ref}$, so

$$H(s) = \frac{V_{ref}}{V_o} = 1 \quad (11)$$

The pulse width modulator is a voltage comparator actually. v_c is the output signal of error amplifier and Compensation network. It is compared with clock signal, then the comparator putout a pulse with the duty cycle is d , that control two main switch of the converter. The transfer function of the PWM comparator is

$$F_m(s) = \frac{\hat{d}(s)}{\hat{v}_c(s)} = \frac{1}{V_m} \quad (12)$$

B. PID Compensator

The most important component of the design is compensator in control circuit. Transfer function of the main circuit is $G_{vd}(s)$, if there is no compensator, the transfer function $G_c(s) = 1$, put it into formula (13), the open-loop transfer function of the system. Then we can get the open-loop transfer function (14) of the system without compensator.

$$T(s) = \frac{H(s)G_c(s)G_{vd}(s)}{V_m} \quad (13)$$

$$T(s) = \frac{G_{vd}(s)}{V_m} = 23.7 \times \frac{\left(1 + \frac{s}{500000}\right) \left(1 - \frac{s}{307433}\right)}{1 + \frac{s}{58.42} + \left(\frac{s}{4502}\right)^2} \quad (14)$$

And $T(0) = 23.7 \Rightarrow 27.5dB$. We can calculate out the crossover frequency $w_{c1} = 1383rad/s$.

$f_{c1} = \frac{w_{c1}}{2\pi} = 220.2Hz$, and the phase margin is 92° . It

can be seen that, the crossover frequency is too low, response speed of the system is too slow, and the phase margin is too large. These are disadvantaged for else dynamic characteristics. So the compensation network is necessary. The PID compensator not only has the advantage that can improve the steady performance of system, but also has the more superiority to improve system dynamic performance. So we chose the PID compensation network, which is shown in fig.4.

Fig.4. PID compensator in this paper

Form the fig.4, we can get the transfer function of the PID compensator, which is formula (15). Then the system Open-loop transfer function after compensated is formula (16).

$$G_c(s) = G_{c0} \frac{\left(1 + \frac{s}{w_z}\right) \left(1 + \frac{w_L}{s}\right)}{\left(1 + \frac{s}{w_{p1}}\right) \left(1 + \frac{s}{w_{p2}}\right)} \quad (15)$$

$$T_c(s) = G_c(s)T(s) = \frac{G_{c0}G_{vd0}}{V_m} \times \frac{\left(1 + \frac{s}{w_z}\right) \left(1 + \frac{w_L}{s}\right) \left(1 + \frac{s}{w_{z2}}\right) \left(1 - \frac{s}{w_{z3}}\right)}{\left(1 + \frac{s}{w_{p1}}\right) \left(1 + \frac{s}{w_{p2}}\right) \left[1 + \frac{s}{Qw_0} + \left(\frac{s}{w_0}\right)^2\right]} \quad (16)$$

The next step is to calculate parameters of the Compensation network. If we make $w_{p1} = w_{z2} = 5 \times 10^5 rad/s$, to cancel the zero point which was caused by output capacitance (ESR), then the

open-loop transfer function is formula (17) now.

$$T_c(s) = G_c(s)T(s) = \frac{G_{c0}G_{vd0}}{V_m} \frac{(1+\frac{s}{w_z})(1+\frac{w_L}{s})(1-\frac{s}{w_{z3}})}{(1+\frac{s}{w_{p2}})[1+\frac{s}{Qw_0}+(\frac{s}{w_0})^2]} \quad (17)$$

If we make crossover frequency f_c of open-loop transfer function after compensation, to one fifth of the switching frequency f_s , that is $f_c = f_s/5 = 50/5 = 10kHz$, and suppose that phase margin is 45° , $\varphi_m = 45^\circ$, then zero-point frequency and pole frequency respectively are

$$f_z = f_c \sqrt{\frac{1 - \sin \varphi_m}{1 + \sin \varphi_m}} = 10 \times \sqrt{\frac{1 - \sin 45^\circ}{1 + \sin 45^\circ}} = 4.142kHz$$

$$f_{p2} = f_c \sqrt{\frac{1 + \sin \varphi_m}{1 - \sin \varphi_m}} = 10 \times \sqrt{\frac{1 + \sin 45^\circ}{1 - \sin 45^\circ}} = 24.142kHz$$

$$f_L = \frac{f_c}{10} = 1kHz$$

and make

From $|T_c(jw_c)|_{dB} = |T(jw_c)G_c(jw_c)|_{dB} = 0dB$, we can get

$$|G_c(jw_c)|_{dB} = 33dB \Rightarrow 44.8$$

Then

$$G_{c0} = |G_c(jw_c)| \frac{\sqrt{1 + (f_c/f_{p1})^2} \sqrt{1 + (f_c/f_{p2})^2}}{\sqrt{1 + (f_c/f_z)^2} \sqrt{1 + (f_L/f_c)^2}} \quad (18)$$

$$= 18.6$$

$$G_c(s) = \frac{18.6(1 + \frac{s}{26011})(1 + \frac{6280}{s})}{(1 + \frac{s}{500000})(1 + \frac{s}{151611})} \quad (19)$$

$$T_{c0} = \frac{G_{c0}G_{vd0}}{V_m} = 23.7 \times 18.6 = 439.1 \Rightarrow 52.8dB$$

$$T_c(s) = \frac{439.1(1 + \frac{s}{26011})(1 + \frac{6280}{s})(1 - \frac{s}{307433})}{(1 + \frac{s}{151611})[1 + \frac{s}{58.42} + (\frac{s}{4502})^2]} \quad (20)$$

$$\text{From } w_{p1} = \frac{1}{R_1 C_1} = 5 \times 10^5 \text{ rad/s},$$

$$w_L = \frac{1}{R_2 C_2} = 6280 \text{ rad/s},$$

$$w_{p2} = \frac{C_2 + C_3}{R_2 C_2 C_3} = 151611 \text{ rad/s}$$

We can get $R_1 = 100\Omega$, $C_1 = 20nF$, $R_2 = 35.3k\Omega$, $C_2 = 4.5nF$, $R_3 = 1.82k\Omega$, and $C_3 = 0.195nF$, respectively. And they meet the following formula.

$$G_{c0} = \frac{R_2 C_2}{R_3 (C_2 + C_3)} = 18.6$$

IV. SIMULATION AND EXPERIMENTAL RESULTS

The small-signal of open-loop and close-loop Boost converter is simulated by using MATLAB, to verify the accuracy of modeling which is proposed in this paper. The Bodes of open-loop transfer function and input-to-output transfer function are shown in fig. 5 (a), (b), respectively. Fig. 6 shows the step response waveform of control-to-output in open-loop and close-loop circuit, respectively. From the figure; we know that, crossover frequency and phase margin of open-loop transfer function in the system are more reasonable after compensation. And has decreased steady-state error in low frequency, decreased amplitude of input-to-output transfer function and output impedance in low and medium frequency, strengthened the disturbance of inhibitory ability, and improved the transient response speed and stability.

(a) The open-loop transfer function (The broken line and real line stand for situations of before and after compensation, respectively).

(b) The input-to-output transfer function (The broken line and real line stand for situations of open-loop and close-loop circuits, respectively)

Fig.5. The small-signal transfer function of Boost converter

Fig.6. The step response waveform of control-to-output in open-loop and close-loop circuit.(The broken line and real line stand for situations of before and after compensation, respectively)

V. CONCLUSION

Based on the average model of switching elements method, an average of the equivalent circuit method, and the idea of energy conservation law, we take Boost converter for example to study the average model of the basic non-ideal converter, which is added voltage control feedback loop in DCM circuit. And we improve the model by using the duty ratio constraint; derive the open-loop transfer function and closed-loop transfer function of the model. Both to be verified the correctness of the model building, under the MATLAB simulation tools. In this paper, we have studied the modeling of voltage-control Boost converter considered parasitic parameters in DCM. The model is useful and intuitionistic, and its physical significance is clear.

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