An Investigation on Zero Cutoff Frequency in a Rectangular Waveguide Periodically Loaded with MNG Metamaterial

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Abstract— In this paper the propagation of electromagnetic wave inside a waveguide which is periodically loaded with anisotropic metamaterial with transversal negative permeability and positive permittivity (MNG) is analyzed. Using Maxwell's equations and Floquet's theorem, dispersion equation of the structure is derived and discussed. The obtained equation is general and valid for propagating and attenuating modes in both anisotropic MNG and air medias. Our study is mainly concentrated on the existence of the zero cutoff frequency in a rectangular waveguide which is periodically loaded with MNG metamaterial. All simulations are done for two different thicknesses of MNG layers and the achieved results are compared with the results of a hollow waveguide. Results indicate that the electromagnetic dispersion curve, zero cutoff frequency, stopband and passband intervals can be highly affected by the permeability and the thickness of the layers.

Index Terms—MNG metamaterial, zero cutoff frequency

I. INTRODUCTION

In recent years analysis of electromagnetic wave propagation in a rectangular waveguide filled with metamaterial has been widely investigated. These materials represent some unusual properties like: anticutoff behavior, backward wave propagation [1-7] and near zero or zero cutoff frequency [8] in some frequency intervals. The above mentioned properties are usually observable in double negative (DNG) and/or single negative (SNG) media. These kinds of medias can be implemented using split ring resonators (SRRs), complementary split ring resonators (CSRRs), thin wires, spiral, triangular shaped rings and etc.

Studying of anisotropic metamaterials is of great significance because of two main reasons: firstly, they exhibit left-handed behavior in some specific directions of propagation and secondly, their fabrication process is much easier comparing to isotropic metamaterial [9]. Here a periodically loaded waveguide with lossless anisotropic MNG metamaterial is theoretically analyzed. The waveguide is excited by TE_{10} mode. Using Maxwell's equations and Floquet's theorem, the dispersion equation is derived. The effect of having different negative values for transversal μ on the wave propagation is discussed using the resulted equation. This can be useful for analysis and better understanding of wave propagation in periodic structures made by metamaterials. This analysis can be useful and applicable for determination and control of pass band or stop-band intervals in filter design. It is worth mentioning

that the zero cutoff frequency of this waveguide contrary to fully filled rectangular waveguide with MNG metamaterial, occurs only for specific ranges of negative transversal permeability and some particular thicknesses of MNG layers. However in the other values of negative transversal permeability the nonzero cutoff frequency may lead to miniaturization effect. The results reveal that the cutoff frequency, dispersion curve, stop band and pass band intervals are highly dependent on the transversal negative μ and also the thickness of MNG layer.

II. ANALYSIS OF PROPAGATION PARAMETERS

The rectangular waveguide with transversal dimensions of $a \times a$ is assumed to be periodically loaded by layers of uniaxial anisotropic MNG metamaterial (Fig.1). The constitutive parameters for the MNG layer are stated by the following tensors

$$\bar{\mu} = \mu_0(zz) + \mu_t\mu_0(xx + yy) = \mu_0 \begin{bmatrix} \mu_t & 0 & 0 \\ 0 & \mu_t & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (1a)$$

$$\bar{\epsilon} = \epsilon_0(xx + yy + zz) \quad (1b)$$

Figure 1. Periodic structure with periodicity of d_2

Let us assume that electromagnetic fields in the waveguide can be written as:

$$E(x, y, z) = E(x, y)e^{-\gamma_1 z} \quad (2a)$$

$$H(x, y, z) = H(x, y)e^{-\gamma_1 z} \quad (2b)$$

where γ_1 , the general propagation constant, is equal to γ_1 for air region and γ_2 for MNG metamaterial region. By solving the vector wave equation in a fully filled waveguide with anisotropic metamaterial (3a), the z component of magnetic field (3b) and also the corresponding dispersion relation can be obtained (3c) [7]

$$\nabla \times (\bar{\epsilon}_r^{-1} \cdot \nabla \times H) - \omega^2 \bar{\mu} H = 0 \quad (3a)$$

$$H_z = H_0 \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{a}\right) e^{-\gamma_2 z} \quad (3b)$$

$$\mu_z = \frac{\mu_t}{k_0^2 \mu_t + \gamma_2^2} \left(\frac{\pi}{a}\right)^2 \quad (3c)$$

By exciting the periodic waveguide and after applying the continuity of tangential electric and magnetic field components at the interface of MNG and air media ($z = d_1$),

the following equations are obtained

$$E_{y \text{ MNG}} = (A_1 e^{-\gamma_2 d_1} + A_2 e^{\gamma_2 d_1}) \sin\left(\frac{\pi x}{a}\right) \quad (4 \text{ a})$$

$$E_{y \text{ air}} = (A_3 e^{-\gamma_1 d_1} + A_4 e^{\gamma_1 d_1}) \sin\left(\frac{\pi x}{a}\right) \quad (4 \text{ b})$$

$$\frac{\partial E_{y \text{ MNG}}}{j\omega\mu_t\mu_0 \partial z} = j \frac{\gamma_1}{\omega\mu_t\mu_0} (A_1 e^{-\gamma_2 d_1} - A_2 e^{\gamma_2 d_1}) \sin\left(\frac{\pi x}{a}\right) \quad (4 \text{ c})$$

$$\frac{\partial E_{y \text{ air}}}{j\omega\mu_0 \partial z} = j \frac{\gamma_2}{\omega\mu_0} (A_3 e^{-\gamma_1 d_1} - A_4 e^{\gamma_1 d_1}) \sin\left(\frac{\pi x}{a}\right) \quad (4 \text{ d})$$

By applying Floquet's theorem on the transversal fields components at $z = d_2$, results in the following relations :

$$A_1 e^{-\gamma_f d_2} + A_2 e^{-\gamma_f d_2} = A_3 e^{-\gamma_1 d_2} + A_4 e^{\gamma_1 d_2} \quad (5 \text{ a})$$

$$\frac{-\gamma_2 A_1 e^{-\gamma_f d_2}}{\mu_t} + \frac{\gamma_2 A_2 e^{-\gamma_f d_2}}{\mu_t} = -\gamma_1 A_3 e^{-\gamma_1 d_2} + \gamma_1 A_4 e^{\gamma_1 d_2} \quad (5 \text{ b})$$

where, γ_f is the propagation constant for the periodic structure. To have a unique solution for the equations (4)-(5) the determinant of the coefficients matrix should vanish.

$$e^{-2\gamma_f d_2} + e^{-\gamma_f d_2} \left[\frac{-1}{\gamma_1 \gamma_2} \left(\frac{\gamma_2^2}{\mu_t} + \gamma_1^2 \mu_t \right) \sinh(\gamma_1 (d_2 - d_1)) \sinh(\gamma_2 d_1) - \frac{2j}{\gamma_1 \gamma_2} \cosh(\gamma_1 (d_2 - d_1)) \cosh(\gamma_2 d_1) \right] + 1 = 0 \quad (6)$$

By considering the expression in bracket as $D(f)$ the above determinant can be written as (7)

$$e^{-2\gamma_f d_2} + D(f) e^{-\gamma_f d_2} + 1 = 0 \quad (7)$$

According to this equation $D(f)$ is obtained as (8)

$$D(f) = -2 \cosh(\gamma_f d_2) \quad (8)$$

This solution is valid in both MNG and air media not only in the propagating conditions (when γ_i is imaginary) but also for the evanescent case (when γ_i is real).

Further, if $|D| < 2$, γ_f becomes imaginary which implies a propagating mode for the periodic structure and if $|D| > 2$, γ_f becomes real and it leads to an evanescent mode.

III. SIMULATION RESULTS AND DISCUSSION

In this section parametric study of the structure is presented. This shows that the geometrical parameter and also the transversal μ can highly affect the propagation characteristics of the considered periodic structure. The obtained results are then compared to the results of a hollow waveguide.

A. Effect of layer's thickness (d_1)

Figure. 2 shows dispersion curve and $D(f)$ diagram of hollow waveguide ($\mu_t = 1$) in the frequency region of 0-100 GHz. The transversal dimension of the waveguide is 15×15 (mm).

The propagation constant and $D(f)$ diagram of the periodic structure are depicted in Fig.3. It is obvious that stop band frequency range for $d_1 = 5$ (mm) is smaller than $d_1 = 8$ (mm). Therefore the cutoff frequency can be controlled by thickness of MNG layer (the cutoff frequency for $d_1 = 5$ mm and $d_1 = 8$ mm are 2.1 GHz and 4.86 GHz respectively).

Figure 2. A hollow waveguide (i.e $\mu_t = 1$), (a) $D(f)$ (b) Dispersion diagram

Figure 3. Combined structure for $\mu_t = -3$, (a) $D(f)$ (b) Dispersion diagram

Also the pass band frequency range for $d_1 = 5$ (mm) is wider than $d_1 = 8$ (mm). The propagation constant and $D(f)$ diagram of the periodic structure for $\mu_t = -0.5$ are depicted in Fig.4. For further investigation, all above simulations are done for transversal permeability $\mu_t = -0.5$ and $\mu_t = -3$ (Fig.5). As it is shown in Fig.5 there is an obvious difference between the obtained results. For example the structure with $\mu_t = -0.5$ has zero cutoff frequency for $d_1 = 8$ (mm) but cutoff frequency $f_c = 10$ GHz for $d_1 = 5$ (mm) (Fig.5-a)

Figure 4. Combined structure (a) $D(f)$ (b) Dispersion diagram in 0-20 GHz for $\mu_t = -0.5$

Figure 5. Combined structure (a) Dispersion diagram in 0-100 GHz for $\mu_t = -0.5$ (b) Dispersion diagram in 0-100 GHz for $\mu_t = -3$

B. Effect of transversal μ (μ_t)

This parameter has strong effect on the cutoff frequency for different layer's thicknesses. Fig.6, exhibits dependency of cutoff frequency on transversal permeability.

Figure 6. Cutoff frequency of periodic structure versus the value of transversal μ for two different thicknesses of the layers

In the case of $d_1 = 5(\text{mm})$, the cutoff frequency has slow variation versus transversal negative permeability for $\mu_t \leq -2.88$. However it has zero cutoff frequency for higher values of μ_t . For $d_1 = 8(\text{mm})$, the cutoff frequency is always zero. Therefore, propagation characteristics are dependent on the transversal permeability only for some specific layer's thicknesses.

IV. CONCLUSION

The analysis of the rectangular waveguide loaded with anisotropic MNG metamaterial is investigated in this paper. The different behavior in comparison to a hollow structure is discussed. Furthermore; dispersion relation and cutoff frequency curve versus geometrical and magnetic parameter are simulated. Based on these characteristics, the considered structure depending on anisotropic MNG material and layer's thickness can exhibit various cutoff frequencies, stopband and passband regions.

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