Abstract—The bifurcation characterized by a phase transition, ie a temperature where the material structure changes qualitatively exists in a number of dielectrics. The Landau model for interpreting this kind of bifurcation, the degree of nonlinearity is controlled by temperature and allows the realization of nonlinear capacitors. An RLC circuit realized by such capacitors present chaotic solutions. We propose to study the nonlinear behavior by linearization in the sense of least squares.

Index Terms—Model Landau, phase transition, capacitor nonlinear bifurcation optimal linearization,

I. INTRODUCTION

The study of nonlinear systems has led physicists to the discovery of a large number of systems with complex behaviours and strange. These behaviours are similar to random behaviour although the systems are nonlinear deterministic have been explored in mathematics by new topological methods. The experiment, measurements and numerical simulations have shown that a large number of physical systems undergo bifurcation and chaos, beginning with mechanical systems the simplest: A system of multiple clocks with several equilibrium positions. The chaos was just noticed in the circuits nonlinear aero elastic systems, systems electro magneto mechanic, etc ...

Indeed, a sequence of bifurcations in reducing turbulence was proposed by Landau. Rule and taken were the first to demonstrate that a strange attractor can be established from a finite sequence of bifurcation and creates turbulent models.

II. NONLINEAR CAPACITOR CONTROLLED BY THE TEMPERATURE

A nonlinear capacitor can be found for example in a varicap diode. However, there are dielectric can be controlled by temperature, allowing the realization of nonlinear capacitor. We present the thermodynamic potential of the dielectric as a function of P and T (which represents a size difference of atoms, may be the degree of order, or a size travel etc ...). If considers the phase transition at a given pressure, the function of the thermodynamic potential can be written:

\[ \Phi(P,T,\zeta) = A(T) = a(T - T_c) \] (1)

Where \( T_c \) is the temperature of transition phase and \( a = \left. \frac{\partial A}{\partial T} \right|_{T=T_c} \), a constant in the phase transition model.

The dielectric energy per unit volume is written

\[ G = \frac{A}{2} D^2 + \frac{C}{4} D^4 \] (2)

Where \( C \) is a constant which depends on the temperature of transition phase, it is assumed positive. Since the electric field in material media checks

\[ dG = EdD \] (3)

the desired relation between \( D \) and \( E \) is written

\[ E = AD + BD^3 \] (4)

Equation (1) characterizing the material in the transition phase, can also describe the equilibrium positions. The free energy of the dielectric has the following extrema

1) \( G \) presents a minimum: Where \( T > T_c \) for \( D = 0 \).
2) \( G \) presents two minima: Where \( T < T_c \) for \( D = \pm \sqrt{-\frac{A}{B}} \).
3) \( G \) presents one maximum : When \( T < T_c \), for \( D = 0 \).

We spend so continuously from the lowest symmetric state to the most symmetric state. (1)

Fig.1: The energy of dielectric per unit volume for different temperatures.
III. CIRCUITS MADE BY A NON-LINEAR CAPACITOR

A. Non-linear RLC circuit

Over the past two decades research on the complex behavior of nonlinear electronic circuits modeled by nonlinear equations has advanced rapidly. Chua's circuit occupies a special place, however, the nonlinearity in the circuit is resistive. The realization of an RLC circuit with nonlinear capacitor allows visualizing solutions and then understanding the dynamic behavior of the circuit. The state equation governing the system is as follows

\[
\frac{d^2 \Phi}{dt^2} + R \frac{d \Phi}{dt} + \frac{e}{L} A_0 (T - T_c) \frac{d \Phi}{dt} + \frac{e}{L S} B D^3 = \frac{U}{L S} \sin \omega t \, . \tag{5}
\]

Assume that the nonlinear capacitor is a capacitor which \(e\) is the thickness, \(S\) represents the surface. By solving the system (5) for:

\[
R = 1000 \Omega, \quad L = 1.8 H, \quad S = 0.0004 m^2, \quad A_0 = 3.18 e 7, \quad B = 6.5 e 11, \quad e = 0.002 m, \quad T_C = 23^\circ \, .5
\]

We note that:

1. For a temperature lower than the temperature of phase transition solutions converge to an equilibrium point different from the origin while the origin is unstable.
2. For a temperature above the transition temperature of phase solutions converge to an equilibrium point at the origin.
3. At the temperature of phase transition we note that the solutions converge to a limit cycle.

We here merely to draw the solutions to the temperature phase transition figure (4) and Figure (5)

\[
\Phi(t) = \frac{U}{L S} \sin \omega t
\]

B. Non-linear circuit LC

The model of nonlinear LC circuit is as follows

\[
\dot{D} + \frac{R}{L} D + \frac{e}{L S} A_0 (T - T_c) D + \frac{e}{L S} B D^3 = \frac{U}{L S} \sin \omega t \, . \tag{6}
\]

We note that this model has a similarity with the Duffing oscillator in some areas of the temperature, like the previous case, there are three regions:

1) The temperature is below the critical temperature: The circuit behavior is similar to the Duffing oscillator. (Figure (6))

2) At the critical temperature (temperature of phase transition): the circuit has periodic solutions but controlled by the term non-self.

3) The temperature is above the critical temperature: the solutions have a center despite the existence of a term non-autonomous (Figure (7)).
C. Non-linear circuit RC

The model describing the circuit is as follows

\[
\begin{align*}
\dot{D} &= -\frac{e}{SR} A_0 (T - T_c) D - \frac{e}{SR} BD^3 \\
D(0) &= D_0
\end{align*}
\]  

We propose to study a model of the circuit system to the linearized free pitchfork bifurcation. The linearization method is the optimal derivation proposed by T. Benouaz and O. Arino [8] [9] [10] [11] [13] [14.. It is involved in the system (7), a system of form

\[
\begin{align*}
\dot{D} &= AD \\
D(0) &= D_0
\end{align*}
\]  

The procedure is based on minimizing the following functional (with \( D(t) = x(t) \))

\[
G(A) = \int_0^{+\infty} \left\| F(x(t)) - Ax(t) \right\| dt
\]

(9)

Along a given solution, starting from a point \( x_0 \) and going to the equilibrium as time tends to infinity. The derivation of (9) with respect to \( A \) gives

\[
A = \left[ \int_0^{+\infty} \left[ F(x(t)) x(t) \right] dt \right]^{-1} \times \left[ \int_0^{+\infty} \left[ x(t) x(t) \right] dt \right]^{-1}
\]

(10)

For the calculation of the matrix, the authors proposed an iterative method which can be summarized as follows (Assume that the spectrum of consecutive matrices is strictly negative real part)

1) \( A_0 \leftarrow DF(x_0) \).

2) \( j \leftarrow 1 \).

3) \( A_j \leftarrow \int_0^{+\infty} \left[ F(e^{A_{j-1}}t) e^{A_{j-1}}t \right] dt \times \left[ \int_0^{+\infty} \left[ e^{A_{j-1}}t e^{A_{j-1}}t \right] dt \right]^{-1} \)

4) if \( \left\| A_j - A_{j-1} \right\| > \varepsilon \).

\( j \leftarrow j + 1 \)

Go To 3.

If No

Optimal derivative \( \tilde{A} \leftarrow A_j \)

End.

We propose to study its behaviour when the temperature varies. The system has three equilibrium points, putting \( \delta = \frac{A_0(T - T_c)}{B} \) it obtains by the optimal derivative the following linear systems

Near the origin \( \left( \frac{dD}{dt} = a_1 D = \left( \delta - \frac{D_0^2}{2} \right) D \right) \)  

(11)

In the vicinity of \( D_{2eq} = \sqrt[3]{\delta} \).

\[
\frac{dD}{dt} = a_2 D = 2 \left[ -\delta - \frac{D_0^2}{4} - 2D_0 \sqrt[3]{\delta} \right] D
\]

(12)

In the vicinity of \( D_{3eq} = -\sqrt[3]{\delta} \).

\[
\frac{dD}{dt} = a_3 D = 2 \left[ -\delta + \frac{D_0^2}{4} + 2D_0 \sqrt[3]{\delta} \right] D
\]

(13)

Indeed, to a temperature below the transition temperature phase the system has three equilibrium points, stability of each depends on the temperature, the problem is the value of phase transition, the method derivation provides an optimal balance unstable initially and then to conclude that the system undergoes a supercritical pitchfork bifurcation in the transition phase, Figures 8, 9 and 10 show the vector fields representing the electric induction state equilibrium. Before the temperature of phase transition temperature phase transition and after the temperature of phase transition.

Figure 8: Vector field representing the electric induction near equilibrium at a temperature below the temperature of phase transition.
IV. CONCLUSION

The model of phase transition Landau helps explain the qualitative changes in the circuits made with a capacitor nonlinear behaviour of solutions can be varied form and undergoes several bifurcations. A sequence of bifurcation can be reduced to chaotic solutions.

A similar study may be to develop superconducting materials. In addition, the temperature of phase transition, the qualitative behaviour of solutions is ignored (since the state of the material is ignored a theoretical point of view). The linearized model proposed allows to know the nature of equilibrium in the value of bifurcation with great precision.

REFERENCES


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