Non-ideal Models and Simulation of Boost Converters Operating in DCM

Guang-jun Xie, *Senior Member, IACSIT*, Hai-bin Fang and Xin Cheng

Abstract—Various aspects for the averaged modelling of the Boost converters operating in the discontinuous conduction mode (DCM) are studied. The parasitics of the Boost converter are also considered. A duty ratio constraint that defines the diode conduction interval is identified to be the key for accurate prediction of high-frequency behaviour. A new duty-ratio constraint is proposed that leads to the full-order averaged models of DCM converters. Numerical analyses and experimental measurements confirm that the new models correctly predict the small-signal responses up to the half of switching frequency and are more accurate than previous models. Moreover, the analytical results are included to show the origin of the high-frequency pole in DCM operation and to explain why the full-order model is capable of accurately predicting it.

Index Terms—Discontinuous Conduction Mode, Boost Converters, Duty Ratio Constraint

I. INTRODUCTION

For the high stability and efficiency of DC-DC switching converter, it has been applied to the fields of mobile communications, industrial equipment, home appliances, and so on. The analyses and design of the switch converter already becomes the key points of the research. However, the pulse-width modulation (PWM) switching converters are strongly non-linear time-varying circuits. It is difficult to accurately find their analytical solutions. So, to find a suitable model for simulation of the DC-DC switching power supply design is an important role.

DC-DC converter has two basic operating modes, namely, continuous conduction mode (CCM) and discontinuous conduction mode. In this paper, the Boost DC-DC converter is taken for example, the average circuit method is used to study its non-ideal model operating in DCM. In the proposed methodology, the so-called duty-ratio constraint is used to make improvement in the model. Thereafter, the calculated transfer-functions for steady-state and dynamic small-signal characteristics are also obtained.

The DCM operation of PWM converters differs from CCM operation by an additional time interval in each

switching cycle during which an inductor current is clamped to zero (or a constant when there are multiple energy storage elements). Therefore, the DCM operation typically occurs in the DC-DC converters at light load which means a low load current. Many efforts have been made in the past decades to model DCM converters[1-6], these models fall into two categories, i.e. linear models and non-linear models. Non-linear model can be also called full-order averaged model [7] which shows great improvement over the linear model. Conduction losses based on the switch-cell and parasitics are included in the non-linear model.

II. DUTY-RATIO CONSTRAINT

The inductor current waveform of Boost converter in DCM is shown in Fig. 1.

Fig.1. The inductor current waveform of Boost converter in DCM

A duty-ratio constraint defining the dependency of d_2 on other variables is identified to be the key to accurately predict the high-frequency behavior. In the conventional state-space averaging method, a volt-second balance relation of the inductor is used to define the duty-ratio constraint [8]. For the Boost topology, volt-second balance over a switching cycle implies

$$d_2 = \frac{v_i}{v_c - v_i} d_1 \quad (1)$$

In the conventional state-space averaging method, using this expression can derive average state equation of ideal converter.

$$\frac{d\bar{i}_L}{dt} = 0 \quad (2)$$

$$\frac{d\bar{v}_c}{dt} = \frac{v_i}{\bar{v}_c} \frac{\bar{i}_L}{C} - \frac{\bar{v}_c}{RC} \quad (3)$$

In equation (2), $\bar{i}_L$ is a linear function, however, in reality

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Gang-jun Xie is with School of Electronic Science and Applied Physics, Hefei University of Technology, Hefei 230009, Anhui, China (corresponding author, e-mail: gjxie8005@hfut.edu.cn).

Hai-bin Fang is with School of Electronic Science and Applied Physics, Hefei University of Technology, Hefei 230009, Anhui, China.

Xin Cheng is with School of Electronic Science and Applied Physics, Hefei University of Technology, Hefei 230009, Anhui, China.

$\bar{i}_L$ is a non-linear function, $d\bar{i}_L/dt$ is relevant with the state variables, $\bar{i}_L$ and $\bar{v}_c$. Equation (1) yields a degenerate model where the dynamics of the inductor current disappear. Since the inductor current is no longer a state variable, this averaged model fails to predict the high-frequency behavior of converters.

In the following, a new definition of d_2 is derived as

$$\bar{i}_L = \frac{i_{\max}}{2}(d_1 + d_2) \quad (4)$$

$$\text{And } i_{\max} = \frac{v_i}{L} d_1 T_s \quad (5)$$

Taking equation (5) into equation (4) can get the solution

$$d_2 = \frac{2L\bar{i}_L}{d_1 T_s v_i} \quad (6)$$

Comparing with equation (1), the equation (6) will derive a full order averaged model which can correctly predict the high frequency responses as well as dc and low frequency behavior of DCM converters.

III. NEW NON-IDEAL MODEL FOR BOOST CONVERTER

Fig.2. The non-ideal Boost converter

An essential step in constructing the dynamic averaged-switch model is to identify the three-terminal network of the switched-inductor cell as shown in Fig. 2. Here, the parasitics are decoupled to form an ideal part of the switched-inductor cell (a, p and c) for which a corrected full-order model based on the fast-state dynamics is considered. Then, the parasitics are placed outside the averaged cell and appropriately modified to account for energy dissipation.

Two switching inductance parasitics are equalled to the slip road under the principle of conservation of energy, and then the average model of switching elements method is used, the switching elements are replaced by a current-controlled current source, the passive switching elements are replaced by a voltage-controlled voltage source, shown in Fig. 3.

Fig.3. The large-signal averaging model of non-ideal Boost converter in DCM

$$R_E = \frac{4}{3(D_1 + D_2)} \left(R_L + \frac{D_1 R_{on}}{D_1 + D_2} + \frac{D_2 R_F}{D_1 + D_2} \right) \quad (7)$$

$$V_E = \frac{D_2}{D_1 + D_2} V_F \quad (8)$$

Considering both the dc and the small-signal ac components of switch current, duty cycle, and switch voltage, there are $d_1 = D_1 + \hat{d}_1$, $\bar{i}_L = I_L + \hat{i}_L$, $v_i = V_i + \hat{v}_i$, $\bar{v}_{ap} = V_{ap} + \hat{v}_{ap}$, $\bar{v}_{zp} = V_{zp} + \hat{v}_{zp}$. For $|\hat{d}_1| \ll D_1$, $|\hat{i}_L| \ll I_L$, $|\hat{v}_i| \ll V_i$, $|\hat{v}_{ap}| \ll V_{ap}$,

$|\hat{v}_{zp}| \ll V_{zp}$, the following results can be deduced.

$$\begin{aligned} \frac{d\bar{i}_L}{d_1 + d_2} &= \frac{d_1^2 T_s v_i}{2L} = \frac{(D_1 + \hat{d}_1)^2 (V_i + \hat{v}_i) T_s}{2L} \\ &= \frac{D_1^2 T_s}{2L} \hat{v}_i + \frac{D_1 V_i T_s}{L} \hat{d}_1 = a_1 \hat{v}_i + a_2 \hat{d}_1 \end{aligned} \quad (9)$$

$$\begin{aligned} & d_1 \bar{v}_{ap} + (1 - d_1 - d_2) \bar{v}_{zp} \\ &= \left[1 + \frac{2L(V_E + V_O + I_L R_E) I_L}{D_1 V_i^2 T_s} - \frac{4L I_L}{D_1 V_i T_s} \right] \hat{v}_i \\ &+ \left[-V_O + \frac{2L(V_i - V_E - V_O - I_L R_E) I_L}{D_1^2 V_i T_s} \right] \hat{d}_1 \\ &+ \left[-R_E + \frac{2L R_E I_L}{D_1 V_i T_s} - \frac{2L(V_i - V_E - V_O - I_L R_E)}{D_1 V_i T_s} \right] \hat{i}_L \\ &+ \left[-D_1 - 1 + \frac{2L I_L}{D_1 V_i T_s} \right] \hat{v}_o = b_1 \hat{v}_i + b_2 \hat{d}_1 + b_3 \hat{i}_L + b_4 \hat{v}_o \end{aligned} \quad (10)$$

Based on the above, the small-signal model is given as follow,

Fig.4. The small-signal model of Boost converter in DCM

(1) The input-to-output transfer function is

$$\begin{aligned}
 G_{vi}(s) &= \left. \frac{\hat{v}_o(s)}{\hat{v}_i(s)} \right|_{\hat{d}_1(s)=0} \\
 &= \frac{1 - b_1 - a_1(sL + R_E + b_3)}{1 + b_4 + \frac{(sL + R_E + b_3)(sR_C C + sRC + 1)}{R(sR_C C + 1)}} \\
 &= \frac{k_{1s} + t_1}{t_0 + \frac{(k_{2s} + t_2)(k_{3s} + t_3)}{k_{4s} + t_4}} \\
 &= \frac{k_1 k_{4s}^2 + k_1 t_{4s} + k_4 t_{1s} + t_1 t_4}{k_2 k_{3s}^2 + k_2 t_{3s} + k_3 t_{2s} + k_4 t_{0s} + t_2 t_3 + t_0 t_4} \\
 &= \frac{t_1 t_4}{t_2 t_3 + t_0 t_4} \cdot \frac{(1 + s/w_{z1})(1 + s/w_{z2})}{1 + \frac{s}{Qw_0} + \left(\frac{s}{w_0}\right)^2} \quad (11)
 \end{aligned}$$

(2) The control-to-output transfer function is

$$\begin{aligned}
 G_{vd}(s) &= \left. \frac{\hat{v}_o(s)}{\hat{d}_1(s)} \right|_{\hat{v}_i(s)=0} \\
 &= \frac{t_4 t_5}{t_2 t_3 + t_0 t_4} \cdot \frac{(1 + s/w_{z2})(1 - s/w_{z3})}{1 + \frac{s}{Qw_0} + \left(\frac{s}{w_0}\right)^2} \quad (12)
 \end{aligned}$$

The control-to-output transfer function includes the right-half plane (RHP) zero w_{z3} , in low-frequency it can be neglected, but in high-frequency it makes the phase of $G_{vd}(s)$ increased -90° . As the Fig. 6(b) shows, the new averaged model gives the most accurate response compared to detailed simulation.

(3) The converter input impedance $Z(s)$ is

$$\begin{aligned}
 Z(s) &= \left. \frac{\hat{v}_i(s)}{\hat{i}_i(s)} \right|_{\hat{d}_1(s)=0} = \left. \frac{\hat{v}_i(s)}{\hat{i}_L(s)} \right|_{\hat{d}_1(s)=0} \\
 &= \frac{t_2 t_3 + t_0 t_4}{t_3 t_7 + t_4 t_6} \cdot \frac{1 + \frac{s}{Qw_0} + \left(\frac{s}{w_0}\right)^2}{(1 + s/w_p)} \quad (13)
 \end{aligned}$$

(4) The converter output impedance $Z_{out}(s)$ is

$$\begin{aligned}
 Z_{out}(s) &= \left. \frac{\hat{v}_o(s)}{\hat{i}_{out}(s)} \right|_{\hat{v}_i(s)=0, \hat{d}_1(s)=0} \\
 &= \frac{t_2 t_4}{t_2 t_3 + t_0 t_4} \cdot \frac{(1 + s/w_{z2})(1 + s/w_{z4})}{1 + \frac{s}{Qw_0} + \left(\frac{s}{w_0}\right)^2} \quad (14)
 \end{aligned}$$

IV. SIMULATION AND ANALYSIS

In this paper, the Matlab/Simulink is chosen as the simulation tool. The parameters are, $V_i = 5V$, $V_o = 10V$, $I_o = 0.2A$, $R = 50\Omega$, $L = 19.2\mu H$, $R_L = 3.6m\Omega$, $C = 1000\mu F$, $R_C = 2m\Omega$, $R_{on} = 10m\Omega$, $R_F = 1m\Omega$, $V_F = 0.4V$, $f_s = 50kHz$.

Taking $D_2 = \frac{2LL}{D_1 T_s V_i} - D_1$, $L = \frac{I_o}{D_2} (D_1 + D_2)$ into equation (11)-(14), specific values can be calculated. The magnitude and phase of the corresponding transfer-functions are plotted in Fig. 6, in which, I, II, III correspond to the following conditions respectively, I. $R_E \neq 0$, $R_C \neq 0$, $V_F \neq 0$, $d_2 = \frac{2L\bar{L}}{d_1 T_s V_i} - d_1$, the parasitics of converter and the definition of d_2 in corrected full-order model are considered. II. $R_E \neq 0$, $R_C \neq 0$, $V_F \neq 0$, $d_2 = \frac{v_i}{v_o - v_i} \cdot d_1$, Taking into account the parasitics, but the duty-ratio constraint is not considered. III. $R_E = 0$, $R_C = 0$, $V_F = 0$, $d_2 = \frac{v_i}{v_o - v_i} \cdot d_1$, for the ideal model.

Fig.5. The model of Boost converter in Matlab/Power System Blockset

To get accurate and repeatable measurement points, In Fig. 5, the hardware prototype is built in Matlab/Power System Blockset, using the FFT utility to extract the necessary magnitude and phase information.

(a) $G_{vi}(s)$

(b) $G_{vd}(s)$

(c) $Z(s)$

(d) $Z_{out}(s)$

Fig.6. The magnitude and phase information of the transfer-functions for Boost converter (in which, the solid line, dashed line and dash-dot line, correspond to I, II, III respectively, for example, the (*) marks in (b) show the hardware measurement)

Suppose as expected, the model proposed in this paper matches the hardware prototype more accurately than the other two previous models. The new model correctly predicts the small-signal responses up to one-half of the switching frequency. Considering the parasitics can more accurately reflect the features of actual converter, it is necessary to consider the parasitics for modeling. A new duty-ratio constraint is proposed, based on formal averaging, which

leads to circuit averaged models that are different from previous ones. Numerical simulation and experimental measurement of Boost converter confirm that the new model correctly predicts the small-signal response.

V. CONCLUSION

In this paper, a general approach has been presented to generate the circuit averaged models of DC-DC converters operating in DCM. The proposed method relies on a duty-ratio constraint and the parasitics of circuit elements are considered. It accurately captures the high-frequency converter dynamics. The proposed model has been verified with a hardware prototype and has shown an improvement over several existing models.

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Guang-jun Xie received the B.S. degree and the M.S. degree from Hefei University of Technology, in 1992 and 1995, respectively. He received the Ph.D. degree from University of Science and Technology of China in 2002. He is now a Professor of Hefei University of Technology, and his research interests include IC design, computational intelligence.