

A Comparative Study of Two Blind Source Separation Approaches to Resolve the Multi-Source Limitation of the Nutating Rising-Sun Reticle Based Optical Trackers

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Abstract—One of the most interesting approaches to the blind source separation problem consists in the use of an appropriate contrast function. Higher-order contrast functions have been proved to be efficient for extracting sources. This paper presents an application of blind source separation in case of convolutive mixture based on frequency and time domain approaches. This work deals with the problem of the multisource limitation of the nutating rising-sun reticle based optical trackers. To resolve this problem, a beam splitter through the modification of the optical trackers has been successfully applied for tracking and discrimination of both coherent and incoherent optical sources based on the semi-blind source separation approach. Furthermore, in relation to frequency-domain algorithm, it has been shown that the time domain algorithm is theoretically consistent without any permutation and amplitude indeterminacy problems.

Index Terms—Blind source separation, Contrast functions, Cumulants, Higher order statistics, Rising-sun reticle.

I. INTRODUCTION

Blind source separation (BSS) has been an active research topic during the past decade due to its potential applications in many areas. As a special case, separation of instantaneous mixtures is very successful so far and many approaches have been proposed [1]. However, a more challenging situation is the separation of convolutive mixtures [2]-[7].

Convolutive blind source separation (CBSS) refers to the separation of signals that have been mixed through a dispersive environment using signal processing procedures that do not have specific knowledge of the source properties or the mixing conditions [8]. The aim of the convolutive BSS is to find filters that when applied to $x(t)$ result in new signals $y(t)$, which is a model of the original source signals $s(t)$. The existing algorithms for solving this problem can be mainly divided into two different categories: the algorithms in the time domain and the algorithms in the frequency domain. Recently, a third approach based on time-frequency representation has also been investigated [9]. This approach is based on the implicit source separation in the time-frequency plane, provided that the signals are sparse

in time and frequency. It seems to be very efficient, especially for speech signals.

The time-domain algorithms include information theory solutions, higher-order cumulant solutions, etc. The frequency-domain blind deconvolution algorithms, transform a convolutive mixture problem into multiple instantaneous mixture problems which can be solved by many algorithms. Thomas *et al.* developed blind separation methods for moving average (MA) convolutive mixtures of independent MA processes [2]. Their method consists of time-domain extensions of the FastICA algorithms developed by Hyvarinen and Oja for instantaneous mixtures [1]. They perform a convolutive sphering in order to use parameter-free fast fixed-point algorithms associated with kurtotic or negentropy non-Gaussianity criteria for estimating the source innovation processes [2]. Their algorithm uses signal deflation, which leads to error accumulation in the separated outputs at each separation stage.

Douglas *et al.* derive two spatio-temporal extensions of the well-known FastICA algorithm of Hyvarinen and Oja that are applicable to the convolutive blind source separation task. Their methods employ least-squares whitening along with novel iterative schemes for maintaining paraunitary constraints on the separation system. A chief advantage of their proposed methods is their simple setup; the algorithms do not require significant parameter tuning in order to obtain good performance [3].

Another approach was developed by Castella *et al.* [4]. Inspired of the semi-blind approach, they proposed new contrast functions for blind signal separation which make use of reference signals. The main advantage of their approach consists in the quadratic form of these criteria: the extraction of one source hence reduces to a simple optimization task for which fast and efficient algorithms are available. The separation of the other sources from the mixture is then carried out by an iterative deflation method [6].

Traditional frequency-domain algorithms consider a large number of frequency bins and need to execute blind separation of instantaneous mixture many times, which results in permutation and amplitude indeterminacy problems. Parra *et al.* developed a frequency domain algorithm that is based on multiple de-correlation assuming non-stationary signals and leads to a least-squares objective function which is minimized to obtain the solution [10]. The advantages and disadvantages of the time and frequency domain approaches

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have been compared by Nishikawa *et al.* [4].

The advantage of the time domain approaches can be summarized as: the better independence assumption for full-band signals and possible high convergence near the optimal point. The disadvantages of these approaches are degradation of convergence in strong reverberant environment and need many parameters to be adjusted for each iteration step.

The first advantage of the frequency domain approaches is that the convolutive mixture can be transformed into instantaneous mixture problems for each frequency bin. The second one is that due to the FFT, computations are saved compared to an implementation in the time domain and finally they have faster convergence. The disadvantages of these approaches are the permutation and a scaling ambiguity; need many samples in each frequency band that may cause the independence assumption to fail, in case of circular convolution the separation performance deteriorates.

Gupta *et al.* have experimentally evaluated the behaviors of several CBSS algorithms for mixtures of strong and weak speech signals. Their results indicate that time-domain CBSS methods provide the greatest SIR improvements for weak-amplitude sources. Frequency-domain CBSS methods have a wider range of performance behaviors and generally do not provide as much enhancement for weak-amplitude sources as do time-domain methods [8].

In this paper, an application of blind source separation in case of convolutive mixture is presented. It is shown that the multisource limitation of the nutating rising-sun reticle based optical trackers can in principle be overcome for both coherent and incoherent optical sources by combining BSS theory and appropriate modification of the optical tracker. In our application, measured signals are convolutive combination of the source signals i.e. we deal to a convolutive mixture.

This paper is organized as follows: In the next section we present principles of convolutive mixture. A brief description of the optical modulation theory and a signal model of the modified optical tracker output are presented in section 3. In the section 4, we extend two algorithms that first one is a time domain algorithm and the second one is a frequency domain algorithm for convolutive mixtures [4], [10]. The experimental performance of the used methods and comparison between two methods is presented in section 5 and conclusions are drawn from this investigation in final section.

Throughout the paper, the following notations are used:

- $\langle . \rangle$ Time average operator.
- $|.|$ Absolute value of a variable.
- $cum\{.\}$ The cumulant of any set of random variables.
- $(.)^*$ Complex conjugate of a vector or a matrix.
- $(.)^T$ Transpose of a vector or a matrix.
- $(.)^H$ Hermitian transpose of a vector or a matrix.
- $diag\{.\}$ The diagonalization operator zeros the off-diagonal elements, i.e. $diag(A)_{ij} = \begin{cases} A_{ij}, & i = j \\ 0, & i \neq j \end{cases}$
- $\mathcal{C}\{.\}$ Contrast function.
- $Tr[.]$ Trace of a matrix.
- $\|.\|$ The matrix norm.
- $E\{.\}$ Expectation operator.

- Λ A diagonal matrix.
- “*” Temporal convolution operator.

II. PRINCIPLE

At the discrete time index n , a mixture of N source signals $s(n) = (s_1(n), \dots, s_N(n))$ is received by an array of N sensors. The received signals are denoted as $x(n) = (x_1(n), \dots, x_N(n))$. In many real-world applications, the sources are said to be convolutively (or dynamically) mixed. The mixing channels are assumed to be FIR of length L , and the separation filters are also FIR with length M . The noise-free convolutive mixing model is given as:

$$x(n) = A(n) * s(n) = \sum_{l=0}^{L-1} A(l) s(n-l) \quad (1)$$

where $A(n) = [a_{ij}(n)]_{N \times N}$ is the FIR-filter mixing matrix. We assume that the transfer function matrix of the mixing system $A(z) = \sum_{n=0}^{L-1} A(n) z^{-n}$ is nonsingular on the unit circle of the complex plane, which guarantees that the sources are separable. The separation-system output is given as:

$$y(n) = H(n) * x(n) = \sum_{l=0}^{M-1} H(l) x(n-l) \quad (2)$$

where $H(n) = [h_{ij}(n)]_{N \times N}$ is the separation system. This can be rewritten in the z -domain as:

$$y(z) = G(z) s(z) \quad (3)$$

with $G(z) = H(z) A(z)$. BSS is considered to be successful if the components of the output vector $y(n)$ are permuted and filtered versions of the signal sources in $s(n)$, which means that $G(z) = PD(z)$, where P is a permutation matrix and $D(z)$ is a diagonal transfer function matrix.

III. OPTICAL MODULATION THEORY AND SIGNAL MODEL

The major disadvantage of the reticle trackers has been proven to be sensitivity on the man-made clutters such as flares or jammers. To resolve this problem a beam splitter based modification of the optical trackers has been successfully used for tracking and discrimination of the both coherent and incoherent optical sources [11]. The modified optical tracker is shown in Fig. 1. In the case of trackers that generate FM signal by means of the nutating rising-sun reticle the rtf is given by [11]-[14]:

$$s(r, \varphi, t) = \cos[\omega_0 t - \beta \sin(\Omega_M t - \varphi)] \quad (4)$$

where $s(r, \varphi, t)$ is the reticle transmission function (rtf), r and φ are spatial variables of the rtf ranging from 0 to R and $-\pi$ to π , respectively and Ω_M is the speed of nutation of the reticle. The fundamental frequency of reticle, $\omega_0 = m\Omega_M$ is frequency modulated [12]. β is the magnitude of modulation that is given by [12]:

$$\beta = m(r/r_0) \quad (5)$$

Also, m is the number of clear and opaque pairs. The output signals of the optical tracker x_1 and x_2 are respectively achieved by [11]:

$$\begin{aligned} x_1(t) &= g_{11}(t) * s_1(t) + g_{12}(t) * s_2(t) + g_{13}(t) \\ &\quad * [s_1(t)s_2(t)] \\ x_2(t) &= g_{21}(t) * s_1(t) + g_{22}(t) * s_2(t) + g_{23}(t) \\ &\quad * [s_1(t)s_2(t)] \end{aligned} \quad (6)$$

where $s_i(t), i \in \{1,2\}$ is the rtf of the rising-sun FM reticle. The impulse responses $g_{ij}, i, j \in \{1,2,3\}$ can be identified from following equations [11], [12]:

$$\begin{aligned} g_{11}(t) &= A_1 g_1(t) B_{11}(\lambda, t) \\ g_{12}(t) &= A_1 g_1(t) B_{12}(\lambda, t) \\ g_{13}(t) &= A_1 g_1(t) 2K_1 K_2 \sqrt{B_{11}(\lambda, t) B_{12}(\lambda, t)} \\ &\quad \times Re\{\gamma_{12}(t)\} \\ g_{21}(t) &= A_2 g_2(t) B_{21}(\lambda, t) \\ g_{22}(t) &= A_2 g_2(t) B_{22}(\lambda, t) \\ g_{23}(t) &= A_2 g_2(t) 2K_1 K_2 \sqrt{B_{21}(\lambda, t) B_{22}(\lambda, t)} \\ &\quad \times Re\{\gamma_{12}(t)\} \end{aligned} \quad (7)$$

$$\begin{aligned} B_{11}(\lambda, t) &= \int \tau(\lambda) I_1(\lambda, t) R_1(\lambda) d\lambda \\ B_{12}(\lambda, t) &= \int \tau(\lambda) I_2(\lambda, t) R_1(\lambda) d\lambda \\ B_{21}(\lambda, t) &= \int \rho(\lambda) I_1(\lambda, t) R_2(\lambda) d\lambda \\ B_{22}(\lambda, t) &= \int \rho(\lambda) I_2(\lambda, t) R_2(\lambda) d\lambda \end{aligned}$$

where $\tau(\lambda)$ and $\rho(\lambda)$ are transmission and reflection coefficients of the beam splitter, respectively, $R_i(\lambda), i \in \{1,2\}$ is the i -th detector responsivity, $A_i, i \in \{1,2\}$ is the i -th detector sensing area, $I_i(\lambda, t), i \in \{1,2\}$ is the intensities detected by i -th detector and $g_i, i \in \{1,2\}$ are impulse responses of the i -th selective amplifier. In general case, K_1 and K_2 are complex constants representing path losses. We will assume here that they are real numbers. Finally $\gamma_{12}(t)$ is the mutual degree of coherence and λ is the wavelength.

Equation (6) stands for a convolutive nonlinear mixture. In fact, it has been shown that for nonlinear mixtures the independence of the outputs insures the separation of the sources, provided that the separating system is the mirror structure of the mixing system. Even for a known nonlinear mixing model, creating a system which implements the exact inverse of this model is not straightforward for most nonlinear models. Recently, Deville and Hosseini defined a large class of possibly nonlinear models, i.e. "additive-target mixtures" (ATM), for which this inversion may be achieved thanks to the nonlinear recurrent networks. They further extended this approach to the "extractable-target mixtures" (ETM) that they also initially introduced in [15]. They illustrated these general approaches for two specific classes of mixtures, i.e. linear-quadratic mixtures and quadratic ones. Since the nonlinear BSS algorithms are just designed for the special types of the non-linearity, it is tried to transform this signal model into linear one.

The signal model described Eq. (6) is reduced into the linear one when the optical sources are incoherent i.e. $\gamma_{12}(t) = 0$. Also, if $\gamma_{12}(t) = \text{const}$, the signal model is transformed into the linear one by simple linear band pass filtering.

Szu *et al.* have shown that the non-linear model, in case of $\gamma_{12}(t) = \text{const}$, can be transformed into linear one by applying linear band pass filtering operation on the measured signals x_1 and x_2 [12]. They have shown that transformation into linear one can be obtained if the following inequalities are fulfilled:

$$\begin{aligned} \omega_{ij} + \omega_{ji} &> \omega_{max} \\ |\omega_{ij} - \omega_{ji}| &< \omega_{min} \\ \omega_{min} &> (\omega_{max} / 2) \end{aligned} \quad (8)$$

where, $\omega_{ij}, i, j \in \{1,2\}$ are corner frequencies of the source signals (s_1 and s_2) and ω_{max} and ω_{min} are the corner frequencies of linear band pass filter. Applying linear band pass filter results in the following new model [12]:

$$\begin{aligned} x_1(t) &= \hat{g}_{11}(t) * s_1(t) + \hat{g}_{12}(t) * s_2(t) \\ x_2(t) &= \hat{g}_{21}(t) * s_1(t) + \hat{g}_{22}(t) * s_2(t) \end{aligned} \quad (9)$$

where $\hat{g}_{ij} = h_{BP}(t) * g_{ij}(t), i, j \in \{1,2\}$ and h_{BP} is impulse response of the linear band-pass filter with the corner frequencies ω_{max} and ω_{min} .

In the most general case (partially or totally coherent optical radiation), when $\gamma_{12}(t)$ is some arbitrary function of time, we can introduce additional artificial source signal, $s_3(t) = Re\{\gamma_{12}(t)\}s_1(t)s_2(t)$. We can either use the BSS method developed for underdetermined representation (i.e. more sources than sensors) in order to recover the three source signals from the two measured signals [5]. Also, an additional beam splitter and one additional detector can be added to recover the three unknown source signals on the basis of three measured signals. We can discard the source signal $s_3(t)$ after recovery because it is unnecessary. Since the first two source signals are sub-Gaussian signals, the third source signal can be even Gaussian [14]. Because the signals s_1 and s_2 are the independent FM signals and we know that FM signals are sub-Gaussian, non-Gaussianity of the source signals is fulfilled. It has been shown in [14] that nonsingularity of the mixing matrix is also fulfilled if the beam splitter transmission coefficient satisfies $\tau(\lambda) \neq \text{const}$ over the wavelength region of interest that is fulfilled for real beam splitters. Consequently it has been shown that the conditions necessary for the BSS theory to work is satisfied.

IV. SEPARATING ALGORITHM

A. Time domain algorithm

It is well-known that one of the most interesting approaches to the blind source separation problem consists in the use of an appropriate contrast function.

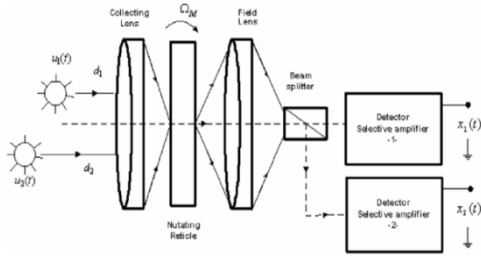


Fig. 1 The modified optical tracker [14]

Basically, a contrast function plays the role of an objective function in the sense that its (global) maximization allows us to solve the problem. In BSS methods, higher-order contrast functions have been proved to be efficient for extracting sources. In this subsection, we describe the method that developed by Castella *et al.* for convolutive mixtures [4]. The main advantage of this approach consists of the quadratic form of these criteria. The extraction of one source hence reduces to a simple optimization task for which fast and efficient algorithms are available. The separation of the other sources from the mixture is then carried out by an iterative deflation method.

A cyclo-stationary process is a signal having statistical properties that vary cyclically with time. In case of cyclo-stationary sources, the contrast function can be used as [4]:

$$C\{y(n)\} = \langle |cum\{y(n), y(n)^*, y(n), y(n)^*\}| \rangle \quad (10)$$

In this case, above contrast function is maximized under the constraint $\langle E\{|y(n)|^2\} \rangle = 1$. This contrast function is valid for both real-valued and complex-valued cyclo-stationary sources. Here, we assume that the mixing model can be denoted by:

$$\forall n \quad x(n) = \sum_k H(k)s(n-k) \quad (11)$$

where, $x(n) = (x_1(n), \dots, x_M(n))$ is the observation vector, $s(n) = (s_1(n), \dots, s_N(n))$ is the source vector and $H(k), k \in \mathbb{Z}$ is the impulse response of the mixing filter with N inputs and M outputs. The contrast functions implemented in the method allow one to extract one source from the mixture. After one source has been restored or if a filtered version of one source has been extracted, one can subtract its contribution in the observation signals. In so doing, the problem of separating N sources from the mixture simplifies to the problem of separating $N - 1$ sources. The so-called “deflation” method in source separation is based on this idea. More precisely, the algorithm is the following one:

- 1) Initialization: first stage ($p = 1$); extraction of the first source $y_1(n), n \in \mathbb{Z}$ from the observations $(x^1(n))_n = (x(n))_n$
- 2) For $p \in \{2, \dots, N\}, y_1(n), n \in \mathbb{Z}$:
 - a) Subtract the contribution of the source extracted at the $(p - 1)$ -th stage; this yields a signal $(x^p(n))_n$ with M components which corresponds to a reduced mixture $N - p + 1$ sources.

- b) Extraction of a p -th source $(y_p(n))_n$ from the modified observations $(x^p(n))_n$.

At stage 2, in order to subtract from the observations the contribution of the $p - 1$ -th source, we compute:

$$x^p(n) = x^{p-1}(n) - \sum_k t^{(p)}(k)y_{p-1}(n-k), n \in \mathbb{Z} \quad (12)$$

where $(t^p(k)), k \in \mathbb{Z}$ is the impulse response of a filter with 1 entry and M outputs. Since the sources are mutually independent, this impulse response is obtained to minimize:

$$\epsilon(t^p(k)) = E\{|x^p(n)|^2\} \quad (13)$$

In practice, the filters have finite impulse response and the above problem amounts to the least square solution of a linear system. Note that deflation methods often lead to an accumulation of errors.

B. Frequency domain algorithm

In this subsection, we describe the method that developed by L. Parra *et al.* [10]. In their method the noisy convolutive mixing model, $x(t) = A * s(t) + n(t)$, is used. As suggested for other source separation frequency domain algorithms, their approach for the convolutive case is to transform the problem into the frequency domain and to solve simultaneously a separation problem for every frequency. Consider the cross-correlations $R_x(t, t + \tau) = \langle x(t)x(t + \tau)^T \rangle$. For stationary signals, the absolute time does not matter and the correlations depend on the relative time, i.e. $R_x(t, t + \tau) = R_x(\tau)$. Denote $R_x(z)$ with the z -transform of $R_x(\tau)$. We can then write:

$$R_x(z) = A(z)\Lambda_s(z)A^H(z) + \Lambda_n(z) \quad (14)$$

where $A(z)$ represents the matrix of z -transforms of the FIR filters $A(\tau)$, and $\Lambda_s(z)$ and $\Lambda_n(z)$ are the z -transform of the auto-correlation of the sources and noise, respectively. They are diagonal due to the independence assumptions.

For practical purposes we have to restrict ourselves to a limited number of sampling points of z . For periodic signals the DFT allows us to express circular convolutions as products such as in (14). However, in (1) and (2) we assumed linear convolutions. A linear convolution can be approximated by a circular convolution if the frame size T of the DFT is much larger than the channel length L . We can then write approximately:

$$x(\omega, t) \approx A(\omega)s(\omega, t) + n(\omega, t) \quad \text{for} \quad L \ll T \quad (15)$$

where $x(\omega, t)$ represents the DFT of the frame of size T starting at t , $[x(t), \dots, x(t + \tau)]$ given by $x(\omega, t) = \sum_{\tau=0}^{T-1} e^{-i2\pi\omega\tau/T} x(t + \tau)$, and corresponding expressions apply for $s(\omega, t)$, $A(\omega)$ and $n(\omega, t)$. For nonstationary signals, the cross-correlation will be time dependent. The sample average of $R_x(\omega, t)$ is given by:

$$\bar{R}_x(\omega, t) = \frac{1}{N} \sum_{n=0}^{N-1} x(\omega, t + nT)x^H(\omega, t + nT) \quad (16)$$

We can then write for such averages:

$$\bar{R}_x(\omega, t) = A(\omega)\Lambda_s(\omega, t)A^H(\omega) + \Lambda_n(\omega, t) \quad (17)$$

If N is sufficiently large we can model $\Lambda_s(\omega, t)$ and $\Lambda_n(\omega, t)$ as diagonal, again due to the independence assumption. For (17) to be linearly independent for different times t it will be necessary that $\Lambda_s(\omega, t)$ changes over time for a given frequency, i.e., the signals are nonstationary.

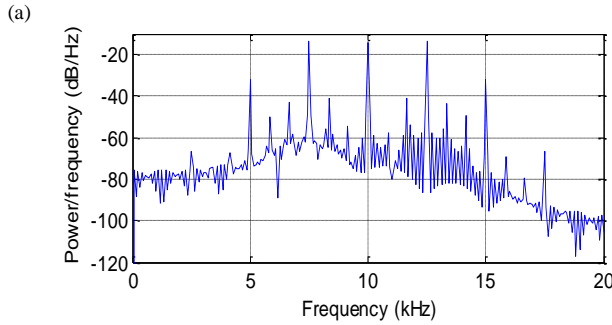
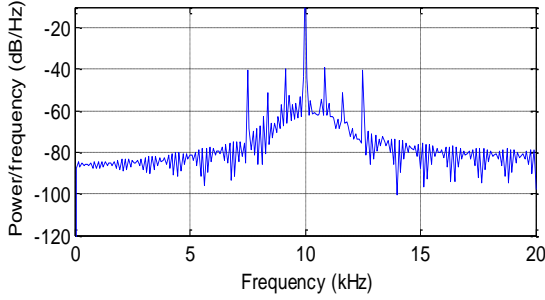


Fig. 2 The power spectrum of the source signals, (a) s_1 and (b) s_2

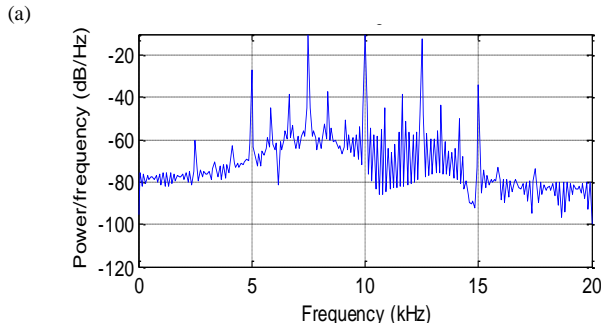
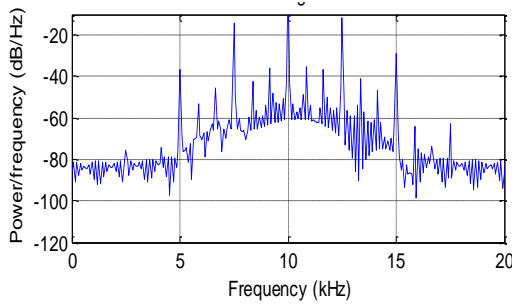


Fig. 3 The power spectrum of the observed signals, (a) x_1 and (b) x_2

In order to directly estimate a stable multi-path backward FIR model such as (2), we should find model sources with

cross-power-spectra satisfying:

$$\Lambda_s(\omega, t) = W(\omega)[\bar{R}_x(\omega, t) - \Lambda_n(\omega, t)]W^T(\omega) \quad (18)$$

A multipath model that satisfies these equations for times simultaneously can be found with a Least Square estimate:

$$E(\omega, k) = W(\omega)[\bar{R}_x(\omega, t) - \Lambda_n(\omega, t)]W^T(\omega) - \Lambda_s(\omega, t) \quad (19)$$

$$\widehat{W}, \widehat{\Lambda}_s, \widehat{\Lambda}_n = \underset{W(\tau)=0, \tau > Q < T}{\operatorname{argmin}} \sum_{\omega=1}^T \sum_{k=1}^K \|E(\omega, k)\|^2 \quad (20)$$

The matrix norm here is the sum of the absolute squared values of every coefficient. Note that $\|E(\omega, k)\|^2 = \operatorname{Tr}[E(\omega, k)E(\omega, k)^T]$. Up to that constraint it would seem the various frequencies $\omega = 1, \dots, T$ represent independent problems. However the solutions $W(\omega)$ are restricted to those filters that have zero time response for $\tau > Q \ll T$. The LS solutions can again be found with a gradient descent algorithm. First the gradient is computed with respect to the complex valued filter coefficients $W(\omega)$ and is discussed their projections onto the subspace of permissible solutions. Therefore the gradients of the LS cost, $J = \sum_{\omega=1}^T \sum_{k=1}^K \|E(\omega, k)\|^2$:

$$\frac{\partial J}{\partial W^*(\omega)} = 2 \sum_{k=1}^K E(\omega, k)W(\omega)[\bar{R}_x(\omega, t) - \Lambda_n(\omega, t)] \quad (21)$$

$$\frac{\partial J}{\partial \Lambda_s^*(\omega, k)} = -\operatorname{diag}[E(\omega, k)] \quad (22)$$

$$\frac{\partial J}{\partial \Lambda_n^*(\omega, k)} = -\operatorname{diag}[W^H(\omega)E(\omega, k)W(\omega)] \quad (23)$$

Again, we can find the minimum with respect to $W(\omega)$, and $\Lambda_n(\omega, k)$ with a constrained gradient descent algorithm using the gradients (21) and (23). The optimal $\Lambda_s(\omega, k)$ for given $W(\omega)$ and $\Lambda_n(\omega, k)$ at every gradient step can be computed explicitly by setting the gradient in (22) to zero, which yields $\widehat{\Lambda}_s(\omega, k) = \operatorname{diag}[W(\omega)\bar{R}_x(\omega, t)W^T(\omega) - \Lambda_n\omega, t]$.

L. Parra *et al.* have found that convergence of the gradient algorithm can be improved substantially if a different adaptation constant is used for every frequency. Note that the gradient terms scale with the square of the signal powers R_x . Because of the signal powers vary considerably across frequency, the gradient terms for different frequencies have very different magnitudes. Normalizing by the powers will therefore scale the gradient to give comparable update steps for different frequencies. This can be achieved easily by defining a weighted cost, $J = \sum_{\omega=1}^T \sum_{k=1}^K m(\omega) \|E(\omega, k)\|^2$. They find good results by choosing straightforward power normalization, $m(\omega) = (\sum_{k=1}^K \|\bar{R}_x(\omega, t)\|^2)^{-1}$. Also the constraint on the filter size Q versus the frequency resolution $\frac{1}{T}$, e.g., $\frac{Q}{T} = 8$, solves the frequency permutation problem.

V. SIMULATION RESULTS

In regard to lack of functional model of modified reticle tracker, observed signals x_1 and x_2 are simulated based on Eqs. (4)-(9). In simulation we assumed that $m = 12$ and $\Omega_M = (20\pi)K\text{rads}^{-1}$ and the optical spots perform circular motion, with radius $r_{01} = 1.2\text{ mm}$ and $r_{02} = 1.5\text{ mm}$, around the center with coordinates $r_1 = 0.1\text{ mm}$ and $r_2 = 0.5\text{ mm}$ relative to the center of the reticle, respectively. The power spectrums of the two frequency modulated (FM) source signals s_1 and s_2 and observed signals x_1 and x_2 are shown in Figs. 2 and 3, respectively. If the time domain algorithm or frequency domain algorithm is applied on the observed signals x_1 and x_2 both IR sources can be discriminated. Power spectrums of the reconstructed signals y_1 and y_2 , are also shown in Figs. 4 and 5.

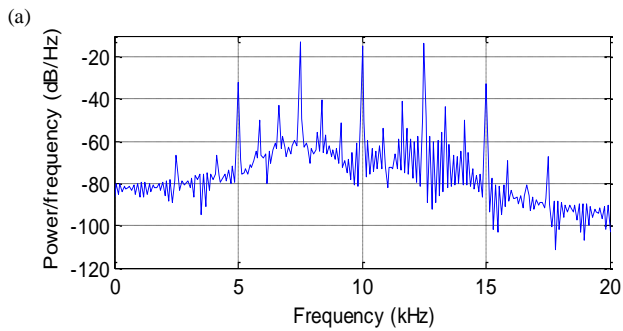
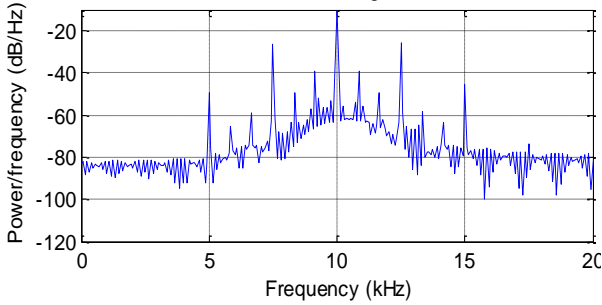


Fig. 4 The power spectrum of the reconstructed signals using time domain algorithm, (a) y_1 and (b) y_2

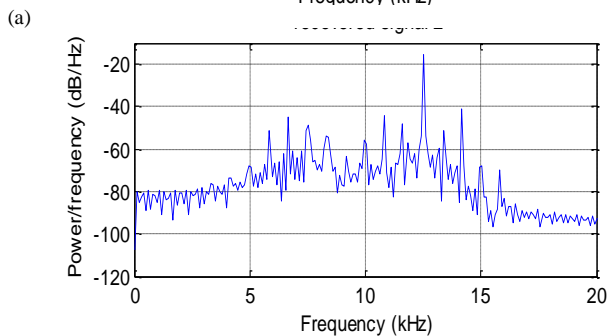
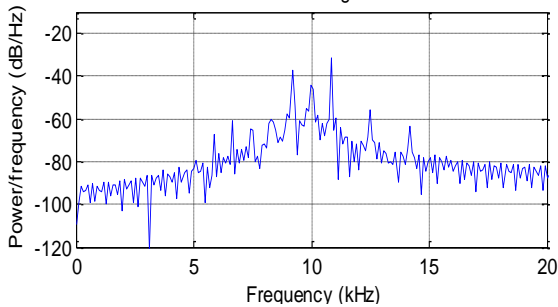


Fig. 5 The power spectrum of the reconstructed signals using frequency domain algorithm, (a) y_1 and (b) y_2

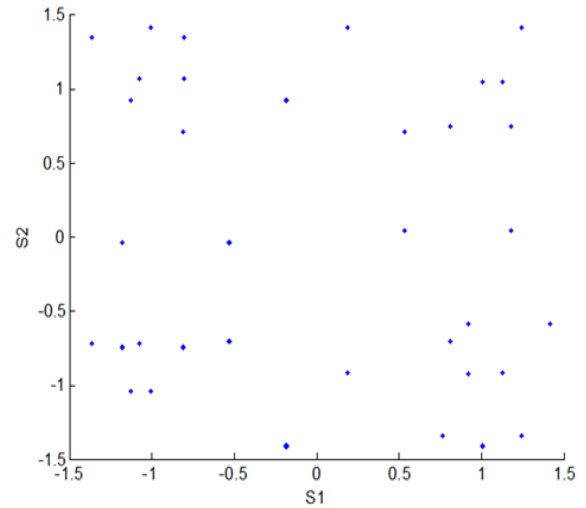


Fig. 6 The joint distributions of s for a linear convolutive mixture

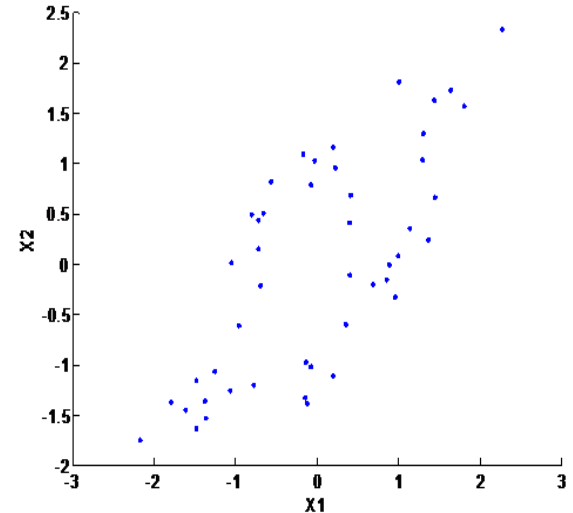


Fig. 7 The joint distributions of x for a linear convolutive mixture

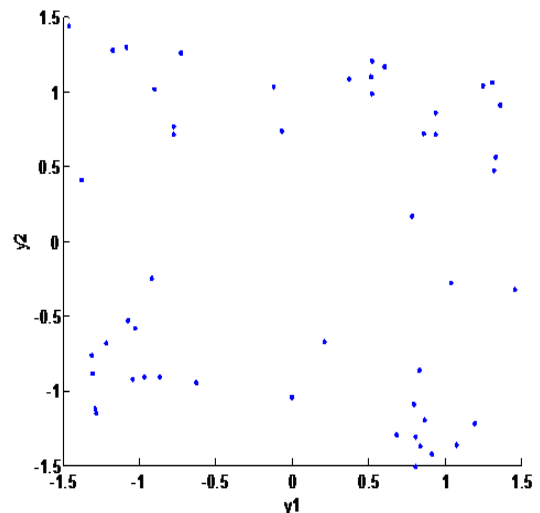


Fig. 8 The joint distributions of y for a linear convolutive mixture using time domain algorithm

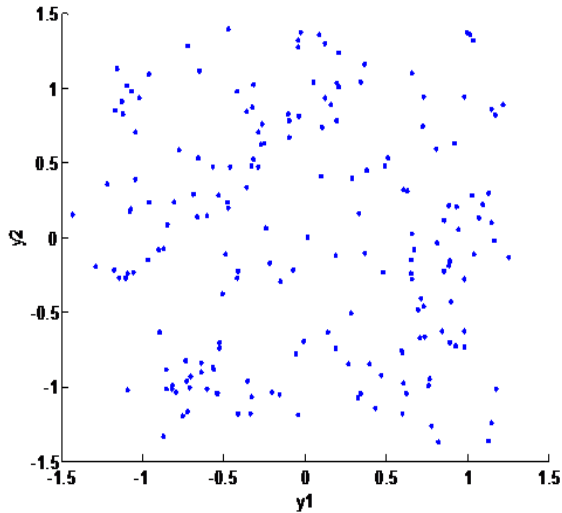


Fig. 9 The joint distributions of y for a linear convolutive mixture using frequency domain algorithm

Figs. 4a and 4b show power spectrums of the reconstructed signals y_1 and y_2 , respectively, using time domain algorithm and Figs. 5a and 5b show power spectrums of the reconstructed signals y_1 and y_2 , respectively, using frequency domain algorithm. Signal y_1 represents reconstructed version of the source signal s_1 while y_2

represents reconstructed version of the source signal s_2 .

The joint distributions of s, x and y are shown in Figs. 6-9, respectively. Fig. 8 shows joint distributions of y using time domain algorithm and Fig. 9 shows joint distributions of y using frequency domain algorithm. These figures shows that, from the independence of s_1 and s_2 , we have $p_{s_1 s_2}(s_1, s_2) = p_{s_1}(s_1)p_{s_2}(s_2)$, and hence the support of $p_{s_1 s_2}(s_1, s_2)$ will be the rectangular region $\{(s_1, s_2) | -1.5 \leq s_1 \leq 1.5, -1.5 \leq s_2 \leq 1.5\}$. In other words, the scatter plot of the source samples forms a rectangular region in the (s_1, s_2) plane. The linear convolutive mapping $x = A * s$ transforms this region into a parallelogram region. The scatter plot of the reconstructed signal samples that forms a rectangular region in the (y_1, y_2) plane proved our claim i.e. the mixed signal is successfully separated.

Fig. 10 and 11 show demodulated signals using time domain and frequency domain algorithms respectively. In each figure, the first one (solid line) obtained after demodulation of the original source signal and the second one (dashed line) obtained after demodulation of the recovered signal. As shown in Figs. 10 and 11, in Fig. 10 two curves are in better agreement than Fig. 11.

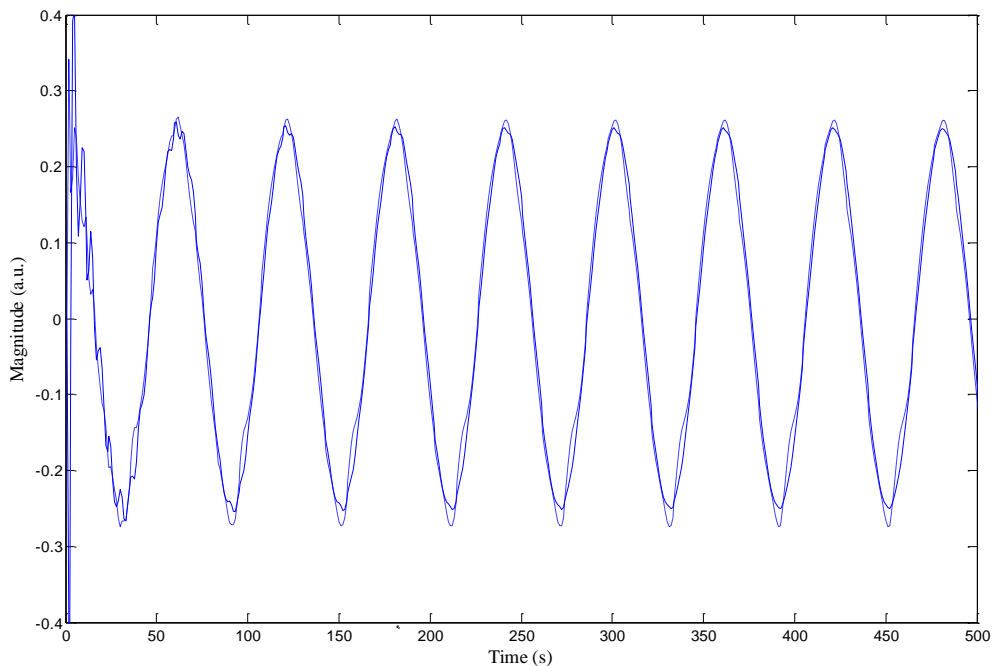


Fig. 10 The demodulated signals. The solid line is original source and the dashed line is recovered source using time domain algorithm

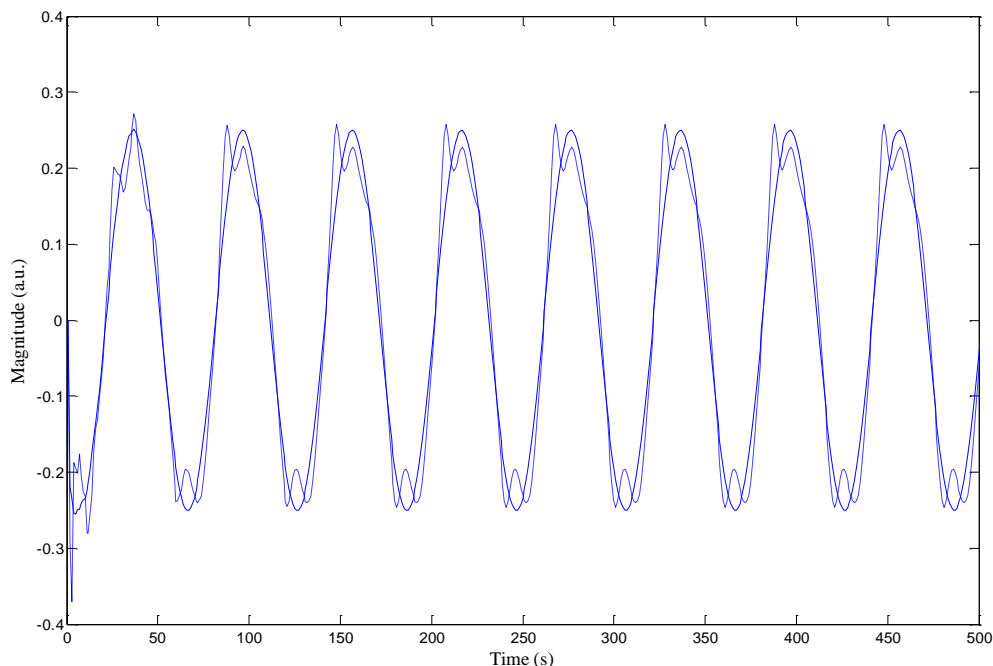


Fig. 11 The demodulated signals. The solid line is original source and the dashed line is recovered source using frequency domain algorithm

VI. CONCLUSION

In this paper, we have experimentally compared the behaviors of two CBSS algorithms for mixtures of IR sources. Comparison the Castella *et al.* approach with the same time domain algorithms indicated that the advantages of this approach are twofold: first, the global maximum of the contrast can be reached. Second, the optimization does not require any iterative gradient-like algorithm which appears to be time consuming because of their slow convergence and the requirement to perform numerous contrast/gradient estimations. It follows that the optimization time is very significantly reduced since the estimation step is performed only once. Comparison this algorithm with the frequency domain algorithm result in better performance, because of frequency domain algorithms have to take into account a permutation and scaling ambiguity at each frequency, furthermore, to provide good results, these methods may require a large number of samples. Also the frequency domain approach exploits a large number of frequency bins, therefore increase the computation cost. Consequently, the multisource limitation of the nutating rising-sun reticle based optical trackers can in principle be overcome for both coherent and incoherent optical sources using time domain and frequency domain BSS approaches.

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