Delay and Slew Metrics for On-Chip VLSI Interconnect

Rajib Kar, Anuran Chattaraj, Aniruddha Chandra, Ashis K. Mal, and Anup K. Bhattacharjee

***Abstract*—In deep sub-micrometer (DSM) regime the on-chip interconnect delay is significantly more dominating than the gate delay. Several approaches have been proposed to capture the interconnect delay accurately and efficiently. By interpreting the impulse response of a linear circuit as a Probability Distribution Function (PDF), Elmore first estimated the interconnect delay. Several other approaches like PRIMO, AWE, *h*-Gamma, WED, D2M etc. have been reported so far, which are shown to be more accurate delay estimation compared to Elmore delay metric. But they suffer from computational complexity when using in the total IC design processes. On the other hand slew rate determines the ability of a device to handle the varying signals. Determination of the slew rate to a good proximity is thus essential for efficient design of high speed CMOS integrated circuits. This in turn estimates the output switching surges in the device. Interconnect slew has become a crucial bottleneck for any high density and high speed VLSI circuits as increased slew results in the increase in delay. Our work presents a closed form formulae for interconnect delay and slew calculation. The proposed metrics are derived by matching circuit moments to the Weibull distribution. This avoids use of complex look-up tables. Experiments validate the effectiveness of our metrics for nets from a real industrial design. We have achieved an average relative error as low as 15% in the delay calculation (compared to the true delay) and 4% in the slew calculation (compared to the true slew value).**

***Index Terms*—Delay calculation, Interconnect, Moment matching, Slew calculation, Weibull Distribution function.**

I. Introduction

As the process technology shrinks into nanometer regime, interconnect delay dominates over the gate delay; hence interconnect delay computation is becoming the crucial bottleneck for both performance and physical design optimization for high speed CMOS integrated circuits. The Elmore Delay [1] which is the first moment of the impulse response provides standard delay estimation for performance driven design applications. Elmore approximated the median of the impulse response (50% delay of the step response) to the mean of the impulse response by noting the similarity between non-negative impulse response and probability density functions. The Elmore delay metric has been incredibly popular because of its simplicity, closed-form and easy to estimate. The inaccuracy of the Elmore metric arises due to the fact that it doesn't consider the resistive shielding effect of the interconnect.

In order to estimate the delay accurately and efficiently several works have been reported so far. Rubenstein et al. [2] proposed a simple closed-form formula for computing the mean of the impulse response of RC interconnect trees. Alpert et al. [3] proposed the D2M metric which is a simple function of the first two circuit moments. The PRIMO [4], *h*-Gamma [5], and WED [6] metrics are based on matching the moments of the impulse response to a particular continuous probability distribution function (PDF). PRIMO and h-Gamma match moments of the impulse response to the Gamma distribution, while WED matches to the Weibull distribution. The approaches proposed in [4-6] require some type of table lookup operation. In order to improve the accuracy of the Elmore delay metric, Asymptotic Waveform Evaluation (AWE) is being proposed [10] by matching the higher order moments of the impulse response. As the technology is shrinking towards the ultra deep sub micrometer (DSM) regime and transistor density in the chip is increasing, the length of the interconnect is getting longer. So efficient and accurate computation of the interconnect delay has become increasingly critical. On the other hand very few approaches [1] [11] have been proposed for interconnect slew calculation. In the nanotechnology age, as ultra deep sub-micron effects continue to wreak havoc on the integrity of the signal, so efficient and accurate computation of the slew metric has become critical.

We present a closed form delay and slew metrics based on the Weibull distribution. Unlike [4]-[6], matching to the Weibull distribution produces closed form formulae and no look-up table is required to compute the delay and slew.

We make the following contributions: A simple delay metric WbD (Weibull delay) and a slew metric WbS (Weibull Slew) are derived using the first two moments of the impulse response. The WbD and WbS metrics can be extended to ramp inputs using the PERI method [8]. The effectiveness of the Weibull metrics is confirmed on nets from an industrial design.

The rest of the paper is organized as follows. Section II, explains the definition and background of the circuit moments. Section III describes the properties of Weibull Distribution Section IV shows the proposed delay metric. Section V describes the proposed slew metric. Section VI presents the simulation results and comparisons with the other established

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matrices. Finally section VII concludes the paper.

II. Basic Theory

Assume that $h(t)$ is the impulse response of a node voltage in an RC circuit. The circuit moments of the impulse response are [7]:

$$m_k = \frac{(-1)^k}{k!}\int_0^\infty t^k h(t)dt \tag{1}$$

where k = 1, 2, 3... and m_k is the k^{th} circuit moment of the impulse response.

The circuit moments can be computed directly as functions of the RC's in time domain e.g., via path tracing algorithm. From [2] the impulse response $h(t)$ satisfies the following conditions:

$$h(t) \geq 0 \quad \text{and} \quad \int_0^\infty h(t)dt = 1 \tag{2}$$

Consequently, the impulse response is a probability distribution function (PDF).

The mean of the impulse response is:

$$\mu = \int_0^\infty th(t)dt \tag{3}$$

Elmore [1] showed that $\mu = -m_1$ and therefore approximated the median (the desired delay) by the mean of the impulse response. We let ED= $\mu = -m_1$, denote the Elmore delay.

The kth central moment is given by:

$$\mu_k = \int_0^\infty (t-\mu)^k h(t)dt \tag{4}$$

The variance (σ^2) of the impulse response can be expressed in terms of the central moments and also the circuit moments [12]:

$$\sigma^2 = \mu_2 = 2m_2 - m_1^2 \tag{5}$$

The key idea behind our delay and slew metrics is to match the mean and variance of the impulse response as available from (1)-(4) to those of the Weibull distribution. This calls for a detailing on the said distribution as will be pointed out in the next section.

III. Properties Of Weibull Distribution

The Weibull distribution is a two-parameter continuous distribution (4-43) [13]. The Weibull distribution is well suited to match the impulse response since both are unimodal and have nonnegative skewness. The Weibull PDF is given by:

$$P(T,\eta,\beta) = \frac{\beta}{\eta}\left(\frac{T}{\eta}\right)^{\beta-1}\exp\left(-\left[\frac{T}{\beta}\right]^{\beta}\right) \tag{6}$$

where $\eta > 0$ and $\beta > 0$ are the scale and shape parameters, respectively. Its cumulative density function (CDF) is given by:

$$D(T,\eta,\beta) = 1 - \exp\left(-\left[\frac{T}{\eta}\right]^{\beta}\right) \tag{7}$$

The expected value (or mean) and the variances are, respectively, given by:

$$E(x) = \eta\Gamma\left(\frac{1}{\beta}+1\right) \tag{8}$$

and,

$$Var(x) = \eta\left\{\Gamma\left(\frac{2}{\beta}+1\right) - \Gamma^2\left(\frac{1}{\beta}+1\right)\right\} \tag{9}$$

IV. Proposed Delay Metric

One can match two common properties of the Weibull distribution and the circuit's impulse response. Note that the mean and variance of the impulse response are $\mu = -m_1$ and $\sigma^2 = 2m_2 - m_1^2$, respectively. Using (8) and (9), by matching the mean and variance yields,

$$\eta\Gamma\left(\frac{1}{\beta}+1\right) = -m_1 \tag{10}$$

and

$$\eta\left\{\Gamma\left(\frac{2}{\beta}+1\right) - \Gamma^2\left(\frac{1}{\beta}+1\right)\right\} = 2m_2 - m_1^2 \tag{11}$$

Let us consider a variable α such that $\alpha = 1/\beta$. We approximate the Gamma function as (6.1.39) [14],

$$\Gamma(az+b) \approx \sqrt{2\pi}\exp(-az)(az)^{az+b-\frac{1}{2}} \tag{12}$$

Now solving (10) and (11) using (12) yields,

$$\alpha = \left(\frac{1}{\ln 16}\right)\ln\left(\frac{4\pi m_2^2}{m_1^4 \ln 16}\right) \tag{13}$$

and,

$$\eta = \frac{m_1}{\Gamma(\alpha+1)} = \frac{m_1}{\sqrt{2\pi}\exp(-\alpha)(\alpha)^{\alpha+1/2}} \tag{14}$$

Quite naturally from (13) we can compute $\beta = 1/\alpha$ as,

$$\beta = \frac{\ln 16}{\ln\left(\frac{4\pi m_2^2}{m_1^4 \ln 16}\right)} \tag{15}$$

Note that the median of the Weibull Distribution is given by

$$M = \eta[\ln(2)]^{1/\beta}$$

(16)

One can verify this by setting $D(T,\eta,\beta) = 0.5$ in (7) and solving for T. Thus, when matching the impulse response the median becomes our 50% delay metric. Substituting η and β values from (14) and (15) in (16) we finally get the delay metric as:

$$WbD = m_1\frac{[\ln(2)]^{\alpha}}{\sqrt{2\pi}\exp(-\alpha)\alpha^{\alpha+1/2}} \tag{17}$$

The above proposed delay function is in closed-form and simple enough in that it involves first two circuit moments only.

V. PROPOSED SLEW METRIC

Let T_{LO} and T_{HI} be 10% and 90% delay points respectively. Matching these points to the CDF yields:

$$0.1 = 1 - \exp\left(-\left[\frac{T_{LO}}{\eta}\right]^{\beta}\right)$$

(18)

and,

$$0.9 = 1 - \exp\left(-\left[\frac{T_{HI}}{\eta}\right]^{\beta}\right)$$

(19)

From (18) and (19), we have

$$T_{LO} = \eta\left[\ln\left(\frac{10}{9}\right)\right]^{1/\beta} \quad (20)$$

and,

$$T_{HI} = \eta[\ln(10)]^{1/\beta}$$

(21)

Finally using (20) and (21) the Weibull slew metric can be written as,

$$WbS = \frac{T_{HI} - T_{LO}}{t_{HI} - t_{LO}} = m_1 \frac{[\ln(10)]^{\alpha} - \left[\ln\left(\frac{10}{9}\right)\right]^{\alpha}}{\sqrt{2\pi t}\exp(-\alpha)\alpha^{\alpha+1/2}} \quad (22)$$

VI. EXPERIMENTAL RESULTS

In order to verify the efficiency of our model, we have extracted 200 routed nets containing 1024 sinks from an industrial ASIC design in 0.13 µm technology. We choose the nets so that the maximum sink delay is at least 20 ps and the delay ratio between closet and furthest sink in the net is less than 0.2. It ensures that each net has at least one near end sink. We classify the 1024 sinks into the following three categories:

1) 511 far-end sinks have delay greater or equal to 75% of the maximum delay to the furthest sink in the net.
2) 342 mid-end sinks which have delay between 25% and 75% of the maximum delay and
3) 171 near-end sinks which have delay less than or equal to 25% of the maximum delay.

For each RC network source we put a driver, where the driver is a voltage source followed by a resister. For each sink we compute the delay and slew using SPICE simulator and measure the relative error of the appropriate metric to the SPICE result. The simulation goal is twofold: first, to compute our delay metric (WbD) with Kahng-Muddu Model (KM) [9] and Elmore Delay (ED [1] and second, to assess our slew metric (WbS) in comparison to [1] and [11]. We call these metrics as ES and BkS, respectively.

Table 1 lists the results with our proposed delay metric (WbD) along with KM and ED. It shows that our approach leads to an average error of less than 15% compared to the true delay value for lower value of driver resistance. From Table 2, we find that our proposed model provides the best slew estimation compared to other approaches and results an average error of less than 4% for higher value of driver resistance.

VII. CONCLUSIONS

We have proposed WbD, a closed form delay metric and WbS, closed form slew metric for RC trees that is a simple function of two moments of the impulse response, for performance optimization. Our metric has the Elmore delay as a theoretical upper bound, but with significantly less error. WbD is more accurate than KM and is indeed remarkably accurate at the near end. WbD has the advantage that its Elmore-like formula may make it more suitable and accurate for optimization purposes. WbS is more accurate than BakS and accurate at the far end nodes.

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Table 1. Comparison for Weibull Delay Metric

Driver resistance = 0Ω						
	Average % Relative error			% Standard Deviation		
Sink	**WbD**	**ED**	**KM**	**WbD**	**ED**	**KM**
Near	45.36	268.72	113.87	33.78	166.37	82.9
Mid	14.72	69.86	15.97	10.38	27.27	12.27
Far	1.03	26.912	1.048	0.952	2.864	1.32
Total	14.51	90.96	29.744	24.83	118.88	59.81
Driver resistance = 100Ω						
	Average % Relative error			% Standard Deviation		
Sink	**WbD**	**ED**	**KM**	**WbD**	**ED**	**KM**
Near	87.6	416.78	197.36	54.31	246.29	131.81
Mid	14.56	64.98	15.11	9.07	22.42	11.58
Far	01.192	29.248	1.32	0.904	3.302	1.032
Total	21.41	114.56	42.08	39.74	183.81	94.3
Driver resistance = 200Ω						
	Average % Relative error			% Standard Deviation		
Sink	**WbD**	**ED**	**KM**	**WbD**	**ED**	**KM**
Near	99.01	366.32	182.72	50.39	182.77	177.7
Mid	11.912	59.62	12.184	6.776	18.496	11.288
Far	1.32	27.736	1.352	0.696	2.936	1.176
Total	22.14	101.28	36.34	31.87	151.28	36.152

Table 2. Comparison for Weibull Slew Metric

Driver resistance = 0Ω						
	Average % Relative error			% Standard Deviation		
Sink	**BakS**	**ES**	**WbS**	**BakS**	**ES**	**WbS**
Near	65.45	786.13	43.72	44.25	615.10	27.38
Mid	11.76	24.27	4.65	7.832	23.62	4.59
Far	9.23	11.23	2.831	6.96	10.30	3.1
Total	21.46	144.12	14.67	34.67	401.1	21.85
Driver resistance = 100Ω						
	Average % Relative error			% Standard Deviation		
Sink	**BakS**	**ES**	**WbS**	**BakS**	**ES**	**WbS**
Near	17.25	143.23	15.34	18.3	98.9	15.66
Mid	12.3	31.2	7.87	10.23	26.56	7.67
Far	9.45	16.6	6.78	7.12	14.43	4.56
Total	10.35	29.4	7.23	10.54	78.65	6.98
Driver resistance = 200Ω						
	Average % Relative error			% Standard Deviation		
Sink	**BakS**	**ES**	**WbS**	**BakS**	**ES**	**WbS**
Near	11.43	84.39	6.5	21.31	105.12	17.82
Mid	13.1	34.68	4.53	7.22	21.29	3.14
Far	8.08	9.76	1.93	6.96	13.14	1.9
Total	9.135	16.269	3.41	10.97	22.11	5.11

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