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State Feedback Controller Design for Exponential Stability of Networked Control System

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***Abstract*—The design of state feedback controller which makes the closed-loop system exponentially stable is studied in this paper for a class of networked control system with time delay and packet dropout. A novel linear discrete time model is proposed which converts the uncertainty of time delay into the uncertainty of parameter matrix. Considering the size of time delay, the networked control system with packet dropout is modeled as an asynchronous dynamical system with rate constraints. Based on TrueTime toolbox, a numerical example is given to illustrate the effectiveness of our results.**

***Index Terms*—networked control system, time delay, state feedback, asynchronous dynamical system.**

I. Introduction

Feedback control systems in which the control loops are closed through a real-time network are called networked control systems (NCSs).[1] As the NCS operates over communicate networks, the time delay and packet dropout which are often the source of instability of NCS are inevitable in the closed-loop control systems.[2][3] Therefore, it is necessary to investigate the stability of NCS with time delay and packet dropout.[4][5]

References [6][7][8] concentrated on the stability of NCS in continuous time domain. However, it is worth pointing out that nearly all the NCSs in many practical systems are controlled by discrete time controllers. And the time delay intervals studied in the above references are subjected to one sampling period. In fact, they are not always within one sampling period. Based on asynchronous dynamical system, references [9][10] paid attention to the stability of NCS, but only packet dropout was considered in those papers. We will extend the method to the NCS with time delay and packet dropout.

The paper is organized as follows. In Section II, the networked control system with uncertain time delay is modeled as a discrete time system with uncertain parameters, and the NCS with time delay and packet dropout is transformed into an asynchronous dynamical system (ADS) with rate constraints. Based on Lyapunov stability theory and LMI approach [11], a state feedback controller is presented to make the system exponentially stable in Section III. A numerical example given in Section IV demonstrates the effectiveness of our results via TrueTime toolbox.

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II. Networked Control System Modeling

Consider the networked control system in Fig.1 including a state feedback controller, where $u(t), \hat{u}(t)$ are the output of controller node and the input of actuator node.

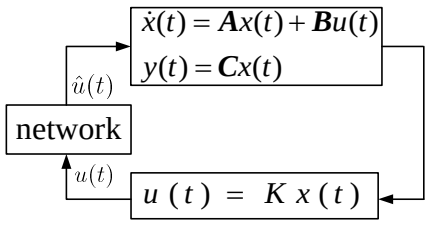

Fig.1. Diagram of NCS with state feedback controller.

We consider the NCS with single-packet transmission, clock-driven sensor, event-driven controller, and event-driven actuator. The sampling period h is fixed and known. The controller node and actuator node communicate with each other via a network. The delay is d_k during the kth sampling period.

In order to improve the system's real-time ability, the data packet will be discarded and be held at previous value once the time delay of this data packet is longer than nh, where n is a given positive number.

Consider a continuous time linear time-invariant system described by:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \tag{2.1}$$

where $x(t), u(t), y(t)$ are the state vector, input vector and the output of the plant respectively. A, B, C are known constant matrices with compatible dimensions. Discretize (2.1) into the following discrete time system during the interval $[kh, kh+h)$, where k is a non-negative integer. We get:

$$\begin{cases} x_{k+1} = \Phi x_k + b_1[\Gamma_{10}(d_k)u_k + \Gamma_{11}(d_k)u_{k-1}] + \cdots \\ \qquad +b_n[\Gamma_{n0}(d_k)u_{k-n+1} + \Gamma_{n1}(d_k)u_{k-n}] \\ (x_{k+1} = x_k \qquad if\ data\ packet\ is\ lost) \\ y_k = Cx_k \end{cases} \tag{2.2}$$

where $\Phi = e^{Ah}$, $\Gamma_{i0}(d_k) = \int_0^{ih-d_k} e^{As}dsB$, $\Gamma_{i1}(d_k) = \int_{ih-d_k}^{h} e^{As}dsB$, $b_i \in \{0,1\}$.

$d_k \in [(i-1)h, ih) \Rightarrow b_i = 1,\ b_j = 0,\ j \neq i,$
$i = 1, \cdots, n, \sum_{i=1}^{n} b_i = 1.$

$d_k \geq nh \Rightarrow b_i = 0,\ i = 1, \cdots, n$ and $x_{k+1} = x_k$.

Consider the above system with state feedback controller $u(t) = Kx(t)$, we have:

$$\begin{cases} x_{k+1} = \Lambda \kappa_k \\ (x_{k+1} = x_k \qquad if\ data\ packet\ is\ lost) \\ y_k = Cx_k \end{cases} \tag{2.3}$$

where:

$\Lambda = [\Phi + b_1\Gamma_{10}(d_k)K, b_1\Gamma_{11}(d_k)K + b_2\Gamma_{20}(d_k)K, \cdots,$
$b_{n-1}\Gamma_{(n-1)1}(d_k)K + b_n\Gamma_{n0}(d_k)K, b_n\Gamma_{n1}(d_k)K],$
$\kappa_k^T = [x_k^T\ x_{k-1}^T\ \cdots\ x_{k-n}^T]$

By the different distribution of time delay in the NCS with or without packet dropout, (2.3) can be viewed as an ADS with rate constraints.

The dynamic behaviors $x_{k+1} = \Lambda\kappa_k$ where $b_i = 1, b_j = 0, j \neq i$ work if the time delays satisfying $d_k \in [ih - h, ih)$, and the probabilities of the events are r_i, where $i = 1, \cdots, n$. The dynamic behavior $x_{k+1} = x_k$ works if the time delay satisfying $d_k \geq nh$ with probabilityr_{n+1}.

It is easy to verify there are $n+1$ dynamic behaviors in NCS, and the probabilities of these events are r_i, $i = 1, \cdots, n+1$ respectively, which satisfy $\sum_{i=1}^{n+1} r_i = 1$.

Define a scalar d^* satisfying the following inequality $[\int_0^{d^*-d_k} e^{As}ds]^T[\int_0^{d^*-d_k} e^{As}ds] \leq I$, the parameters in (2.3) could be transformed as follows:

$$\begin{aligned}\Gamma_{i0}(d_k) &= \int_0^{ih-d^*} e^{As}dsB + \int_{ih-d^*}^{ih-d_k} e^{As}dsB \\ &= \int_0^{ih-d^*} e^{As}dsB + e^{A(ih-d^*)}\int_0^{d^*-d_k} e^{At}dtB\end{aligned}$$

Since $\Gamma_{i1}(d_k) + \Gamma_{i0}(d_k) = \int_0^h e^{A\theta}d\theta B$, we have:

$\Gamma_{i1}(d_k) = \int_{ih-d^*}^{h} e^{As}dsB - e^{A(ih-d^*)}\int_0^{d^*-d_k} e^{At}dtB.$

For convenience of the following proof, we define $\Gamma_0(i) = \int_0^{ih-d^*} e^{As}dsB$, $\Gamma_1(i) = \int_{ih-d^*}^{h} e^{As}dsB$, $D(i) = e^{A(ih-d^*)}$, $F(d_k) = \int_0^{d^*-d_k} e^{At}dt$ and $E = B$, where $F^T(d_k)F(d_k) \leq I$. Then, we get:
$\Gamma_{i0}(d_k) = \Gamma_0(i) + D(i)F(d_k)E$,
$\Gamma_{i1}(d_k) = \Gamma_1(i) - D(i)F(d_k)E.$

III. Design Of State Feedback Controller

To facilitate further research, we present some lemmas as follows.

Lemma 1(Schur complement)[12]

Given constant matrices M_1, M_2, M_3 with compatible dimensions, where $M_1 = M_1^T$ and $M_2 = M_2^T > 0$, then $M_1 + M_3^T M_2^{-1} M_3 < 0$ is equivalent to one of the following two conditions holds:

$$\begin{bmatrix} -M_2 & M_3 \\ M_3^T & M_1 \end{bmatrix} < 0\ or \begin{bmatrix} M_1 & M_3^T \\ M_3 & -M_2 \end{bmatrix} < 0$$

Lemma 2 [13]

For any real matrices Q_1, Q_2, Q_3, Δ_k with compatible dimensions, where $\Delta_k^T\Delta_k \leq I$ and $Q_1 = Q_1^T$, then $Q_1 + Q_3\Delta_k Q_2 + Q_2^T\Delta_k^T Q_3^T < 0$, if and only if there exists a scalar $\epsilon > 0$ such that $Q_1 + \epsilon^{-1}Q_3Q_3^T + \epsilon Q_2^T Q_2 < 0$.

Lemma 3 [14]

Set the events in an asynchronous dynamical system with rate constraints as E_i and r_i are the probabilities of these events respectively, where $i = 1, 2, \cdots, N$. The system is exponentially stable if there exist scalars $\beta_1, \beta_2, \alpha_1, \alpha_2, \cdots, \alpha_N > 0$ and a Lyapunov functional $V_k(E_1, \cdots, E_N)$ such that:

1)$\beta_1\|x\|^2 \leq V_k \leq \beta_2\|x\|^2$
2)$V_{k+1}(E_i) < \alpha_i V_k(E_i)$, where $i = 1, 2, \cdots, N$
3)$\alpha_1^{r_1}\alpha_2^{r_2}\cdots\alpha_N^{r_N} < 1$

Theorem 1

Consider a continuous time linear time-invariant plant described by (2.1) with state feedback controller $u(t) = Kx(t)$, the sampling period h is fixed and known. The probabilities of $d_k \in [ih - h, ih)$, $d_k > nh$ are r_i, $i = 1, 2, \cdots, n+1$ respectively. The data packet will be discarded and be held at previous value once the time delay of this data packet satisfies $d_k \geq nh$. We can design a state feedback controller which makes the closed-loop system exponentially stable if there exist symmetric positive definite matrices $X, Q_1, \cdots, Q_n$, feedback gain matrix Y, and scalars ε, $\alpha_i > 0$, $i = 1, 2, \cdots, n+1$, such that the following inequalities hold. And the feedback gain matrix is $K = YX^{-1}$.

$$\Xi_i = \begin{bmatrix} \Xi_{11} & \Xi_{21}^T \\ \Xi_{21} & \Xi_{22} \end{bmatrix} < 0$$

where $b_i = 1, b_j = 0, j \neq i, i = 1, 2, \cdots, n$

$$\Xi_{n+1} = \begin{bmatrix} X + Q_1 - \alpha_{n+1}X & & & \\ & \ddots & & \\ & & Q_n - \alpha_{n+1}Q_{n-1} & \\ & & & -\alpha_{n+1}Q_n \end{bmatrix} < 0$$

$$\prod_{i=1}^{n+1} \alpha_i^{r_i} < 1$$

where $*$ denotes the corresponding transposed block matrix due to symmetry, I denotes the identity matrix with appropriate dimensions, 0 and blank denote the null matrix with appropriate dimensions.

$$\Xi_{11} = \begin{bmatrix} \varepsilon D(i)D^T(i) - X & \Phi X + b_1\Gamma_0(1)Y & b_2\Gamma_0(2)Y + b_1\Gamma_1(1)Y & \cdots & b_n\Gamma_1(n)Y \\ * & Q_1 - \alpha_i X & & & \vdots \\ * & & Q_n - \alpha_i Q_{n-1} & & \\ \vdots & & & & \\ * & & & \cdots & -\alpha_i Q_n \end{bmatrix}$$

$$\Xi_{21} = \begin{bmatrix} \underbrace{0\cdots0}_{i} & EY & -EY & \underbrace{0\cdots0}_{n-i} \end{bmatrix}$$

$$\Xi_{22} = -\varepsilon I$$

Proof.

Let us consider the following Lyapunov functional $V_k = \sum_{i=0}^{n} x_{k-i}^T P_i x_{k-i}$, where P_i are symmetric positive definite matrices.

1) It is known to all, continuous function $f(x) = x^T P_i x / \|x\|^2$ is bounded in the compact set $\|x\| = 1$, and n is a constant number, that is, there exist

scalars β_1, β_2 satisfying $\beta_1\|x\|^2 \le V_k \le \beta_2\|x\|^2$.

2) If the time delay satisfies $d_k \in [ih-h, ih)$, $i = 1, 2, \cdots, n$, we can get $V_{k+1}(E_i) - \alpha_i V_k(E_i) < 0$ if $\kappa^T \Pi_i \kappa < 0$ holds, where:

$$\Pi_i = \begin{bmatrix} P_1 - \alpha_i P_0 & & & \\ & \ddots & & \\ & & P_n - \alpha_i P_{n-1} & \\ & & & -\alpha_i P_n \end{bmatrix} + \Lambda^T P_0 \Lambda$$

Using the Schur Complement, $\Pi_i < 0$ is equivalent to:

$$\begin{bmatrix} -P_0^{-1} & \Phi + b_1\Gamma_{10}(d_k)K & b_1\Gamma_{11}(d_k)K + b_2\Gamma_{20}(d_k)K & \cdots & b_n\Gamma_{n1}(d_k)K \\ * & P_1 - \alpha_i P_0 & & & \\ * & & \ddots & & \\ * & & & P_n - \alpha_i P_{n-1} & \\ * & & & & -\alpha_i P_n \end{bmatrix} < 0$$

Because of $\Gamma_{i0}(d_k) = \Gamma_0(i) + D(i)F(d_k)E$, $\Gamma_{i1}(d_k) = \Gamma_1(i) - D(i)F(d_k)E$ and $d_k \in [ih-h, ih)$, we can get $b_i = 1$, $b_j = 0, j \ne i$ and the above inequality can be rewritten as:

$$\Sigma + \begin{bmatrix} D(i) \\ 0 \\ \vdots \\ 0 \end{bmatrix} F(d_k)\Theta + \Theta^T F^T(d_k) \begin{bmatrix} D(i) \\ 0 \\ \vdots \\ 0 \end{bmatrix}^T < 0$$

where:

$$\Sigma = \begin{bmatrix} -P_0^{-1} & \Phi + b_1\Gamma_0(1)K & b_1\Gamma_1(1)K + b_2\Gamma_0(2)K & \cdots & b_n\Gamma_1(n)K \\ * & P_1 - \alpha_i P_0 & & & \\ * & & \ddots & & \\ * & & & P_n - \alpha_i P_{n-1} & \\ * & & & & -\alpha_i P_n \end{bmatrix}$$

$$\Theta = \begin{bmatrix} \underbrace{0 \cdots 0}_{i} & EK & -EK & \underbrace{0 \cdots 0}_{n-i} \end{bmatrix}$$

Taking Lemma 2 into account, the above inequality holds if and only if there exits a scalar ε satisfying:

$$\Sigma + \varepsilon \begin{bmatrix} D(i) \\ 0 \\ \vdots \\ 0 \end{bmatrix} \begin{bmatrix} D(i) \\ 0 \\ \vdots \\ 0 \end{bmatrix}^T + \varepsilon^{-1}\Theta^T\Theta < 0$$

By the Schur Complement, the following inequality can be derived:

$$\begin{bmatrix} \varepsilon D(i)D^T(i) - P_0^{-1} & \Phi + b_1\Gamma_0(1)K & b_1\Gamma_1(1)K + b_2\Gamma_0(2)K & \cdots & b_n\Gamma_1(n)K & * \\ * & P_1 - \alpha_i P_0 & & & & * \\ * & & \ddots & & & * \\ * & & & P_n - \alpha_i P_{n-1} & & * \\ * & & & & -\alpha_i P_n & \\ \Theta & & & & & -\varepsilon I \end{bmatrix} < 0$$

Define $X = P_0^{-1}, Y = KX, Q_i = XP_iX, i = 1, \cdots, n$. Pre-and post-multiplying both sides of above matrix by $diag(I, P_0^{-1}, \cdots, P_0^{-1}, I)$, result in $\Xi_i < 0, i = 1, \cdots, n$.

If the time delay satisfies $d_k > nh$, the following matrix inequality is the sufficient condition of $V_{k+1}(E_{n+1}) - \alpha_{n+1}V_k(E_{n+1}) < 0$:

$$\begin{bmatrix} P_0 + P_1 - \alpha_{n+1}P_0 & & & \\ & \ddots & & \\ & & P_n - \alpha_{n+1}P_{n-1} & \\ & & & -\alpha_{n+1}P_n \end{bmatrix} < 0$$

Pre-and post-multiplying both sides of above matrix by $diag(P_0^{-1}, \cdots, P_0^{-1})$, we get $\Xi_{n+1} < 0$.

It is obvious that the matrix inequalities $\Xi_i < 0, i = 1, \cdots, n+1$ imply $V_{k+1}(E_i) - \alpha_i V_k(E_i) < 0$, $i = 1, \cdots, n+1$

3) Since the probabilities of $d_k \in [ih-h, ih)$, $d_k > nh$ are r_i, $i = 1, 2, \cdots, n+1$ respectively. So the probabilities of $\Xi_i < 0$ are r_i, $i = 1, \cdots, n+1$ respectively. Due to $\prod_{i=1}^{n+1} \alpha_i^{r_i} < 1$, the system is exponentially stable. Moreover it is easy to obtain the feedback gain matrix $K = YX^{-1}$.

IV. Example

Consider the system described by (2.1) with parameters as follows:

$$A = \begin{bmatrix} 0 & -2 \\ 5 & -5 \end{bmatrix},$$
$$B = \begin{bmatrix} -0.4 \\ 1 \end{bmatrix},$$
$$C = \begin{bmatrix} 0 & -0.1 \end{bmatrix}.$$

Sampling period of the plant is $h = 0.1$s. The probabilities of $d_k \in [0, h)$, $d_k \in [h, 2h)$, $d_k > 2h$ are 0.6, 0.3, 0.1 respectively.

The plant is transformed to model (2.3), where:

$$\Phi = \begin{bmatrix} 0.9577 & -0.1548 \\ 0.3870 & 0.5708 \end{bmatrix},$$
$$D(1) = \begin{bmatrix} 0.9652 & -0.1430 \\ 0.3575 & 0.6077 \end{bmatrix},$$
$$D(2) = \begin{bmatrix} 0.8690 & -0.2310 \\ 0.5776 & 0.2915 \end{bmatrix},$$

$$\Gamma_0(1) = \begin{bmatrix} -0.0425 \\ 0.0645 \end{bmatrix},$$
$$\Gamma_1(1) = \begin{bmatrix} -0.0053 \\ 0.0044 \end{bmatrix},$$
$$\Gamma_0(2) = \begin{bmatrix} -0.098 \\ 0.0893 \end{bmatrix},$$
$$\Gamma_1(2) = \begin{bmatrix} 0.0507 \\ -0.0204 \end{bmatrix},$$
$$E = \begin{bmatrix} -0.4 \\ 1 \end{bmatrix},$$
$$d^* = 0.01.$$

Applying Theorem 1 where $n = 2$, we can get the following matrix:

$$\Xi_1 = \begin{bmatrix} \varepsilon D(1)D^T(1) - X & \Phi X + \Gamma_0(1)Y & \Gamma_1(1)Y & 0 & 0 \\ * & Q_1 - \alpha_1 X & 0 & 0 & (EY)^T \\ * & * & Q_2 - \alpha_1 Q_1 & 0 & -(EY)^T \\ * & * & * & -\alpha_1 Q_2 & 0 \\ * & * & * & * & -\varepsilon I \end{bmatrix}$$

$$\Xi_2 = \begin{bmatrix} \varepsilon D(2)D^T(2) - X & \Phi X & \Gamma_0(2)Y & \Gamma_1(2)Y & 0 \\ * & Q_1 - \alpha_2 X & 0 & 0 & 0 \\ * & * & Q_2 - \alpha_2 Q_1 & 0 & (EY)^T \\ * & * & * & -\alpha_2 Q_2 & -(EY)^T \\ * & * & * & * & -\varepsilon I \end{bmatrix}$$

$$\Xi_3 = \begin{bmatrix} X + Q_1 - \alpha_3 X & & \\ & Q_2 - \alpha_3 Q_1 & \\ & & -\alpha_3 Q_2 \end{bmatrix}$$

We obtain the following values via the LMI toolbox of Matlab satisfying the matrix inequalities in theorem.

$$X = \begin{bmatrix} 91.7414 & 72.9397 \\ 72.9397 & 153.745 \end{bmatrix},$$
$$Q_1 = \begin{bmatrix} 9.8111 & 10.6107 \\ 10.6107 & 41.0214 \end{bmatrix},$$
$$Q_2 = \begin{bmatrix} 5.5280 & 5.7608 \\ 5.7608 & 22.474 \end{bmatrix},$$
$$Y = \begin{bmatrix} 0.090 & -0.182 \end{bmatrix},$$
$$\varepsilon = 7.8299,$$
$$\alpha_1 = 0.9,$$
$$\alpha_2 = 1.1,$$
$$\alpha_3 = 1.1,$$
$$r_1 = 0.6,$$
$$r_2 = 0.3,$$
$$r_3 = 0.1,$$
$$\alpha_1^{r_1}\alpha_2^{r_2}\alpha_3^{r_3} = 0.9752 < 1.$$

We have feedback gain matrix:

$$K = [0.0031 - 0.0026].$$

Then, the simulation diagram via TrueTime toolbox of the NCS is given in Fig.2. It is assumed that the initial condition of the state is $[-5; -9]$, and the above controller is used, the plant state responses are illustrated in Fig.3.

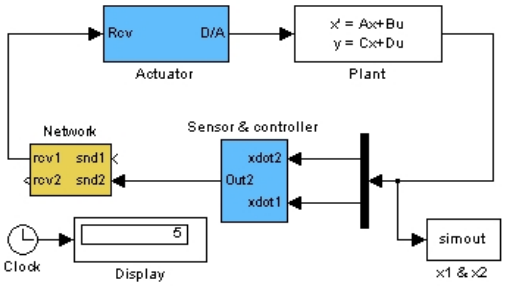

Fig.2. Simulation diagram of the NCS via TrueTime toolbox.

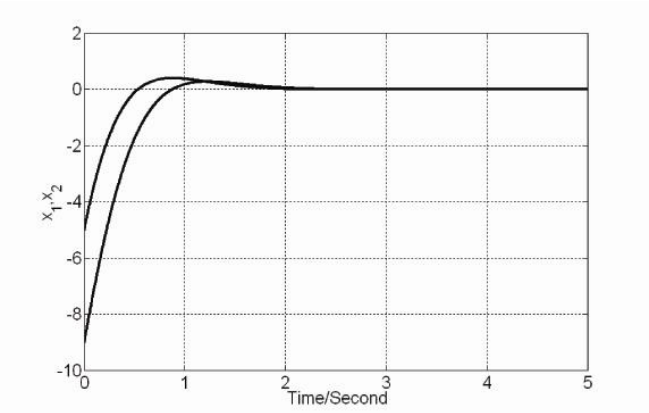

Fig.3. State response curves for the NCS.

V. Conclusion

For a class of NCS with time delay and packet dropout, a novel linear discrete time model has been presented which transforms the uncertainty of time delay into the uncertainty of parameter matrix. Viewing the NCS as an ADS with rate constraints, based on Lyapunov stability theory and LMI approach, we have designed a state feedback controller which makes the closed-loop system exponentially stable. The numerical example has illustrated the effectiveness of our results.

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